MUSICAL ASPECTS OF FRACTAL GEOMETRY

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ABSTRACT

The author will discuss the direction and goals of his research into applications of fractal processes to automated musical composition. Factors relating to the choice of fractal methods, the application of geometric formalisms to music, and the aesthetic implications of self-similarity will be discussed. Tape recorded examples of musical fractals will be played. The report will conclude with a summary of problems encountered and challenges presented to future research in this area.

INTRODUCTION

Fractal geometry has proven itself a remarkable formal model for natural organic and inorganic processes. Among the many applications discovered by the leading researcher in the field, Dr. Benoit Mandelbrot of the I.B.M. Thomas J. Watson Research Centre, and described in his 'casebook' "The Fractal Geometry of Nature" are: the growth of trees and tree like structures; fluid turbulence; stuctural details in Saturn's rings; complex organic spectra; noise and data transmission errors; stock market trends; the structure of coastlines; molecular Brownian motion; and the distribution of matter in galaxies. One of the reasons that fractal geometry is so successful as a model for these phenomenon is a remarkable quality known as 'self-similarity'. Simply stated, this term indicates that similar patterns are repeated on many levels of structure throughout the fractal object.

SELF-SIMILARITY IN MUSIC

Music more than any other art incorporates self-reference as an essential feature; indeed a good case can be argued that a piece of music is entirely and exclusively self-reflexive, acquiring meaning through internal coherence only and not through reference to anything outside itself. Heinrich Schenker developed an analytical system based on processes involved in the perception of pitch and tonality which clearly demonstrates the multi-leveled nature of musical perception and the frequent repetition of significant motivic features at different levels. This form of analysis divides musical perception into foreground, middleground and background, without these, however, being sharply separated from one another - a curious relation to the continuous merging of dimensions involved in fractal geometry. Self-similarity between different levels of structure, frequently seen in the works of the great classical composers, is briefly described in the following passage from Schenker's *Der Freie Satz* :

"As they move toward the foreground, the transformation levels are actually bearers of the developments and are, at the same time, repetitions or parallelisms in the most elevated sense - if we permit ourselves to use the word 'repetition' to describe the movement from transformation level to transformation level. The mysterious concealment of such repetitions is an almost biological means of protection: repetitions thrive better in secret than in the full light of consciousness." (p.18)
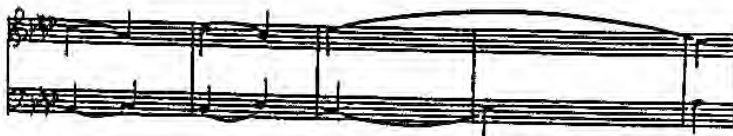
composition the essential motive is a composed out third:

(figure 1)

It is found at several levels, from a background line that guides the overall direction of the phrase:
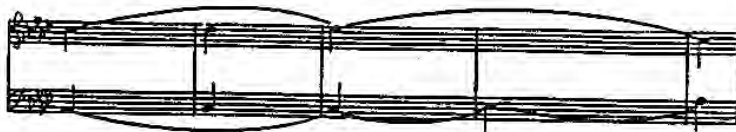
(figure 2)

through the middleground elaborations that fill out the principal line:

(figure 3)

to the elegant foreground melodic ornaments that intensify the musical energy and lend a rather ornate character:

Beethoven Opus 110

(figure 4)

Schenker to be one of the elemental musical shapes *(urlinie)*, arising out of the physical nature of sound through the fundamental musical phenomenon of the overtone series.

Fractal objects possess self-similarity to an astonishing degree. In pure non-random fractals an unlimited number of levels may be strictly self-similar. Practical limits are reached only in the reproduction medium (e.g. graphics resolution) or in human perceptual processes. Fractal objects may also have random aspects, in which case they tend toward a modelling of natural objects and processes. Such techniques have been used with great success in the synthesis of images of natural objects by computer. Some famous example are the fractal mountain scenes synthesized by Richard Voss and reproduced by Mandelbrot (Mandelbrot 1983).

The relationship between musical and fractal structures is clear. Some work in investigating this relationship has already been done, notably the study of music as a scaling (1/f) noise by Voss and Clarke (Voss and Clarke 1975) and the subsequent experiments by Tomasso Bolognesi in pitch sets generated by a fractal process known as *Levy flight* (Bolognesi 1983). Bolognesi's pitch sets are self-similar only in a statistical sense - similar structures are repeated on different scales in an unpredictable fashion, according to the largely random processes he employed. His experiments remain essentially sophisticated techniques of dice-throwing to obtain pitch sets. Of considerably greater interest for the purposes of this project has been the work done by Charles Ames of the State University of New York at Buffalo in recursive programming of musical structures (Ames 1982); and by Gary Kendall of Northwestern University in his imaginative transformation of geometric structures into musical ones (Kendall 1981).

RECURSIVE TRANSPOSITION

Recursion is a very general programming concept which has been widely employed as an elegant

problem solving technique in many areas of computer science. Many fractal processes are in fact most easily implemented recursively. In Douglas Hofstadter's's definition recursion is "nesting, and variations on nesting... Stories inside stories, movies inside movies, paintings inside paintings, even Russian dolls inside Russian dolls". The particular musical application in the context of this research was the ability to specify a set of intervals which would be applied recursively to the entire composition, generating all levels of structure and detail. For practical reasons this program was first implemented employing a table of one hundred elements which together *statistically* specified the interval structure of the composition. Rigidly self-similar structures could be generated by filling the table with a single value throughout; statistically self-similar structures could be generated by the appropriate distribution of values. In the early versions of the program this fractal process could be applied only to pitch and volume aspects of individual notes; durations remain unchanged, which results in the constant patter of short note lengths apparent in *FRACTAL STUDY 1*; an interesting rhythmic structure is generated by the fractal process at the phrase length level, however. The process itself is illustrated graphically below. It consists of a recursive divide by two operation; that is, the transposition process is first applied to the entire composition, which may be several thousand notes long;

(figure 5)

each of the two parts formed is likewise divided into two and the transposition applied;
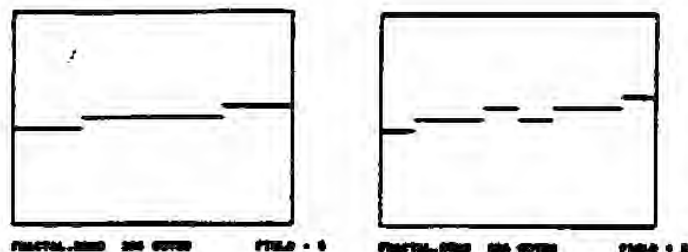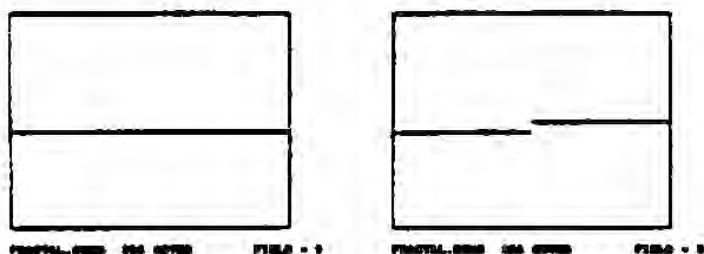

(figure 6)

each of the four resulting sections is divided into two, and so on; the process continues until the limit of resolution is reached at the level of individual notes.
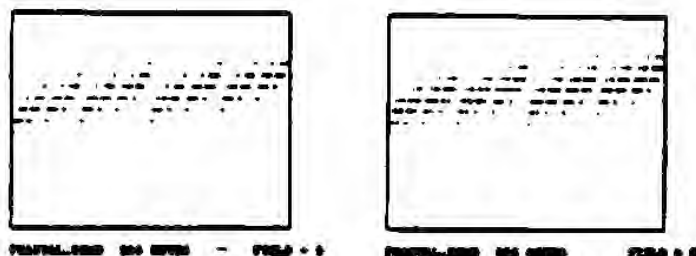
(figure 7)

When the generating interval is, for example, the upward interval of a fifth (as in the examples graphed above), the second half of the composition consists of the first repeated a fifth higher; the first half consists of two periods, the second of which is identical to the first repeated a fifth higher; and so on down to the level of pairs of individual notes. The same process when applied to the dynamic aspects of the piece results in a clearly perceptible structure of frustrated crescendi.

The interesting structures produced by this process were primarily the result of two things: the generating interval pattern chosen and the divisor in the recursive division operation. Experimentation with these resulted in *FRACTAL*

STUDY TWO, which was produced with a divide by three process and a wider range of generating intervals than the earlier work. The recursive divide by three generated a structure reminiscent of classical dance forms and particularly of the rondo, with digressions embedded within statement and re-statement. *FRACTAL JAM* was also produced with this version of the program. It employs a variety of generating interval sets and recursive divisors in order to simulate a rather free improvisatory jazz ensemble.

THE MONSTER CURVES

The above processes are really only special cases of a much broader category of fractals described by Mandelbrot as 'monster curves'. These include the space filling curves whose unusual properties were first described by Koch and Peano at the turn of the century. Mandelbrot has developed a practical theory and nomenclature of these types of fractals which served the needs of this research quite well (Mandelbrot 1983). Implementation of these ideas resulted in a much more general version of the earlier program where transposition patterns could be described precisely and non-statistically, and the recursive divisor could be specified easily. The program was modified to allow the fractal process to operate on the durational relationships between notes as well. In Mandelbrot's terminology we distinguish two elements in the process - the INITIATOR and the GENERATOR. The first is the structure with which one begins; in Mandelbrot's examples usually a triangle, square, or other elementary geometric figure:
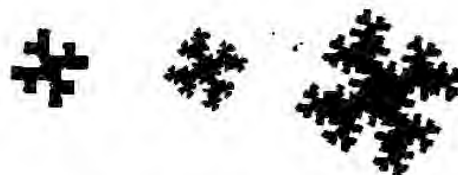
(figure 8)

The generator is the particular pattern that is recursively applied to the orginating structure to produce a fractal object:
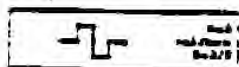
(figure 9)

Mandelbrot defines a generator in terms of the number of sides, N, and their configuration, which is usually given a graphic representation, as above. In the program described, the generator is specified in terms of the number of steps (corresponding to N above) and their relationship is specified as a series of integers representing positive and negative transpositions. The originator, as before, could be any pre-existing composition defined by the means available in MIDIFORTH, but was generally a simple set of several hundred identical notes. The duration problem was solved with a more sophisticated implementation of the core transposition operators, whereby they 'knew' whether they were operating on pitch, volume, articulation or duration data. *FRACTAL DUET*, for synthesized saxophones and harpsichord, was produced with this version of the program.

(figure 10 - Koch curves)

In the preceding examples the contrapuntal relationships between musical parts had not been a part of the generating process. Experiments with contrapuntally related initiators and generators did produce some interesting textures and harmonic progressions, but did not seem to offer the degree of control that was desired.

STRANGE ATTRACTORS

For this reason the research changed direction and turned to a different type of fractal process as offering more promise for coherent relationships between parts. The phenomenon of attractor points in non-linear dynamical systems is an area of investigation that has been thoroughly inter-penetrated by fractal ideas in the last few years (Mandelbrot 1983). It seems to offer some of the first clues to an understanding of previously elusive "chaotic" behaviour, such as that found in fluid turbulence (e.g. the swirls and eddies in a rapidly flowing stream of water). The process itself is

uncomplicated; essentially, iteration of a simple formula results in the formation of "attractor" points within the number plane, areas of relative stability around which other points accumulate. In one well studied system, that generated by:

$$x \longrightarrow lambda\ x(1-x)$$

The behaviour of the system alters dramatically as lambda increases; from a simple two state oscillation at lambda< .86237, the system becomes progressively four state, then eight state, then sixteen, and so on until a limiting value is reached at lambda= .89248, upon which it assumes a "chaotic" behaviour. It should be noted that Mandelbrot has determined that this process results in a fractal dust of dimension approximately .538 (Mandelbrot 1983 p. 195). The range of *lambda* values that produce such behaviour, namely .89248<*lambda*<1.000, is known as the "chaotic region" and abounds in beautiful symmetries and self-similarities.

It should be noted that the pre-chaotic region .75<*lambda*<.89248 also has self-similar aspects. The boundary values at which splitting of the oscillation patterns of the system occur are related by a constant value which is approached more and more closely as the system evolves toward higher numbered stable states. This value was called *delta* by its discoverer, Mitchell Feigenbaum (Feigenbaum 1981). Its value is approximately 4.669201609... Not only this, but the relation of the attractor points to one another is also self-similar, being arranged in a clearly recursive order. This remarkable fact was also discovered by Feigenbaum. He named the ratio *alpha*; its value is approximately 2.502907... In practical terms what this means is that there is a self-similarity of details within an attractor group to the relations between attractor groups. This relates directly, on the one hand to the swirls within swirls that are so characteristic of turbulent motion; and on the other to the interrelationship of different levels of detail apparent in a piece of music.
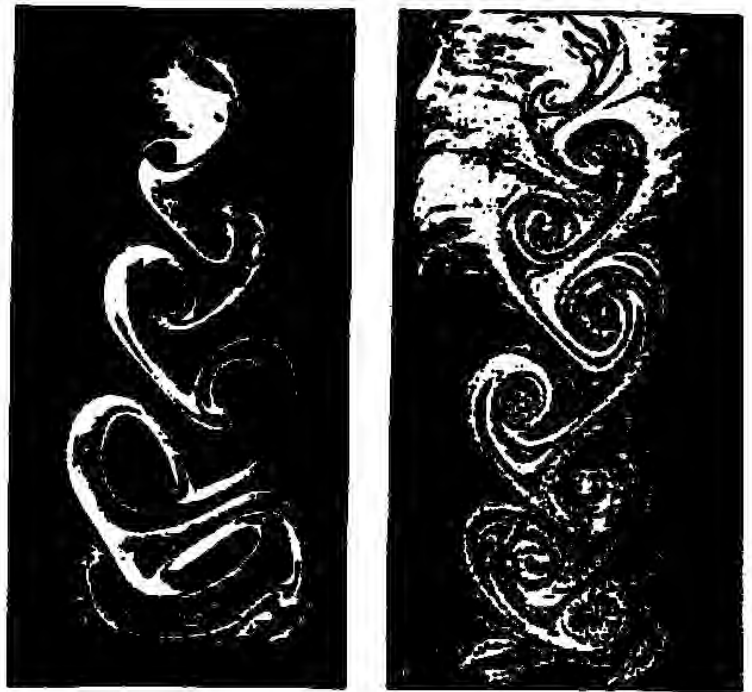
(figure 11 - fluid turbulence)

The region of *lambda* between .89 and 1.00 is the really interesting area however. Not surprisingly it bears a resemblance to the region *lambda*<.89248. The two regions are in fact symmetrical around the chaotic breakpoint at .89248; the symmetry can be clearly seen if one studies the chart in the figure below.
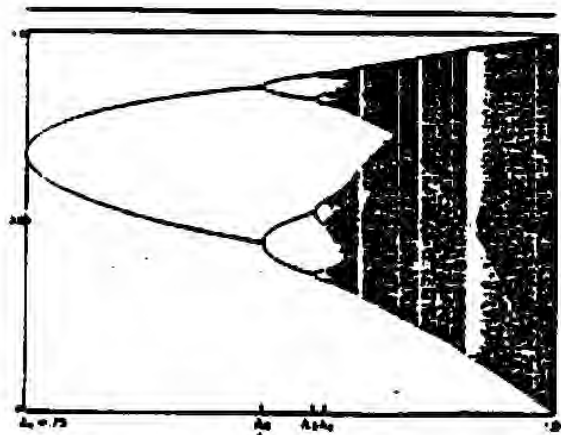
*(figure 12 - transition from cyclic to chaotic behaviour)*

The onset of chaos is clearly marked at *lambda* = .89248; the shaded area to the right is the chaotic region. The symmetry is apparent as one observes the behaviour of x starting at *lambda* = 1.000 and progressing toward *lambda* = .89. At a value of 1.00, x is totally random; it can assume any value within the broad shaded area between 0 and 1. At *lambda* = .92... however, the broad band splits into two, which means that x can assume values in a quasi-periodic fashion somewhere within either of these bands. Continuing to move to the left, a second splitting occurs at *lambda* = .906, at which point x may assume one of four values within the shaded regions. As may be expected this process continues to the limiting point of *lambda* = .89248. Now, even for *lambda* values within the chaotic region attractor points may arise but they do not necessarily do so. The patterns that emerge therefore tend to cycle with some difficulty around stable points. This is the characteristic that seems to lend a 'composed' quality to pitch sequences generated by this method bringing to mind elementary lessons in aesthetics describing 'unity and variety' in a work of art.

In addition to the general structural features just described, the chaotic region contains many peculiar areas (indicated by the white bands in fig 12) where the entire fractal evolution from regular to chaotic motion is reproduced in miniature. The largest band, centered at *lambda* = .9609...., shows a remarkable set of transitions involving a *three* cycle beaviour.

The method of working with these phenomena was quite straightforward. After implementing the function $x \longrightarrow lambda\ x(1-x)$ in the MIDIFORTH computer music environment, a set of pitches determined by the attractor values would be generated - essentially a monophonic line of equal length notes. The pitches could be mapped within a more or less limited range, but volumes, durations, articulations, etc were not affected by the fractal process. Next, after inspecting the generated pitch set, the pitch regions that corresponded to attractor groups would be stripped away using the MIDIFORTH 'filter' facility. Since pitch values falling within any particular attractor group are generally unevenly distributed for *lambda* values within the chaotic region, rhythms of a unique and highly eccentric nature were generated. These, along

with their corresponding pitch sets, can vary greatly in perceived periodicity within the chaotic region, so a large variety of fractal 'styles' was possible. After stripping away notes belonging to particular attractor groups the resulting parts would be orchestrated appropriately and recorded. Thus the musical lines generated are a direct representation of the behaviour of attractor points for the chosen value of *lambda*.

The mimicking of musical intelligence that is apparent in these patterns seems to lend itself well to the satiric mode of expression. The two works produced employing these techniques therefore have a socio-musical aspect that goes beyond the abstract patterning of the previous pieces. *STRANGE ATTRACTIONS* is nominally a study of the turbulence created by a dripping tap, but in actuality it is a statement on the turbulence of musical styles in our time. The sink is the commercial music industry of North America – the whorls and eddies of musical fashion carry us from one piece of flotsam to another until in the end the plug is pulled and the vortex carries us down. In *ROADS TO CHAOS* the fractal division process described above was appropriated as a metaphor for a bleak historical view that sees the evolution of technology as a divisive and self-destructive force. Both pieces employ many references to musical styles as an essential feature beyond the abstract structural schemes drawn from fractal geometry.
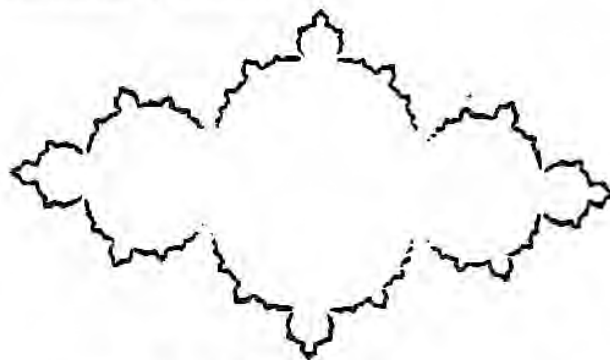
(figure 13 – map of inverse of $x \longrightarrow lambda\ x(1-x)$)

PROBLEMS, AND DIRECTIONS FOR FUTURE WORK

A major problem arises when attempting to apply synthetic (i.e. mechanical) self-similarity to a

musical structure. The fact is that, unlike a work of visual art, the various levels of structure are not perceived in the same way. The problem may be succinctly described as a perceptual discontinuity between structural levels. Foreground events, for example, force themselves directly upon the ear, requiring only the attention of the listener in order to be perceived. Middleground events, on the other hand, require both an active memory and a conscious effort for their recognition, while background events require careful listening, study, and analysis. Another serious problem arises from the temporal nature of the medium itself – it is not uniform and symmetrical, as is the visual field, since later events are affected by those that preceded them. A clear example of this is the different treatment accorded antecedent and consequent phrases of music in everything from folksong to symphony. The fractal processes described above know nothing of such distinctions, although the strange attractor methods can sometimes come up with an amusing (and accidental) imitation. Other problems and challenges arise out of the computer science side of this research. A fully developed programming environment is required to implement the musical 'intelligence' needed to confront these problems. And the work carried out so far has involved a narrow sub-set of the fractal methods described by Mandelbrot; in particular it has not yet touched upon the the most striking and perhaps best known type of fractal, the "Mandelbrot Set".

The harmonic forces active in musical composition are present in these pieces in only a rudimentary form. One logical development of this work would be the ability to specify harmonic structures (in a Schenkerian sense) as well as melodic ones. The harmonic structure might be generated recursively, as with the melodic structures described above; and compositionally related melodic material might be fractally generated in a second set of operations. Such a program would require a much more elaborate set of operators than have currently been employed, since they would have to 'know' about such things as keys and major and minor forms of intervals. It would in fact approach something like the LISP based Schenkerian analysis program written by Smoliar (Smoliar 1980) with a change in emphasis from an analytical system to a generative one.

Another extension of this study might be downward, towards timbral scales of structure. There is no reason, with a general purpose software-oriented digital synthesizer, why related fractal processes could not be used to generate structures at all levels of scale within the composition. The problem of perceptual discontinuity discussed above would become extreme, however, and would surely be the first problem addressed. These questions and others point the direction for further work in automated composition to proceed.

(figure 14 - self-squared dragon)

ACKNOWLEDGEMENTS

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