

Physical Modelling of Musical
Instruments Using Digital
Waveguides:

History, Theory, Practice

Introduction

- Why Physical Modelling?
- History of Waveguide Physical Models
- Mathematics of Waveguide Physical Models, via Data Flow Diagrams
- Demonstration of Yamaha VL synthesizer

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- NOT looped playback of previously recorded sound (e.g. samplers)

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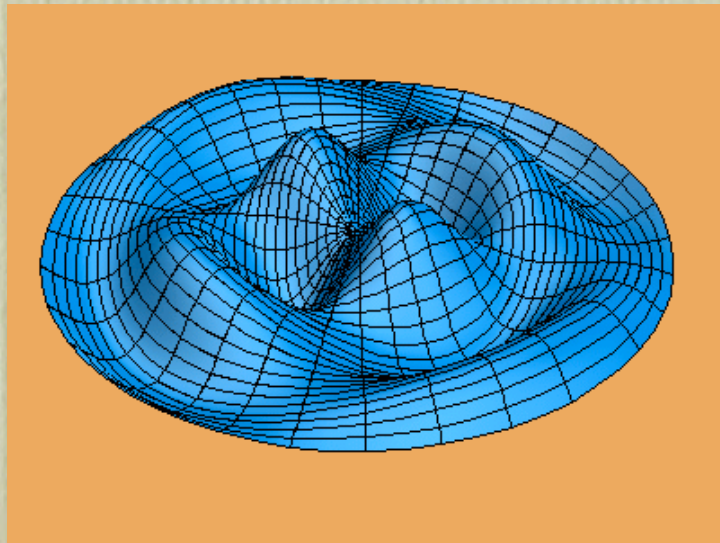
- NOT synthesis by *ad hoc* similarity of sound
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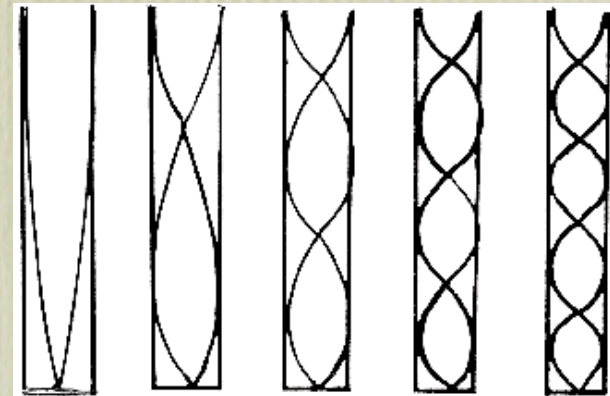
What IS Physical Modelling?

- computer sound synthesis based on physical theory of the vibrating object - string, reed & air column, membrane



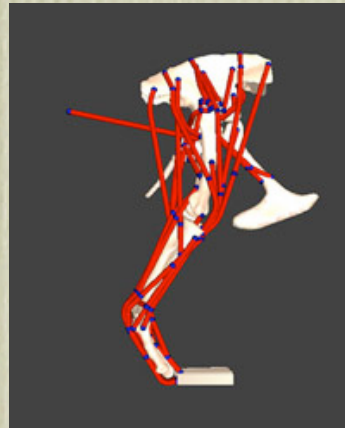
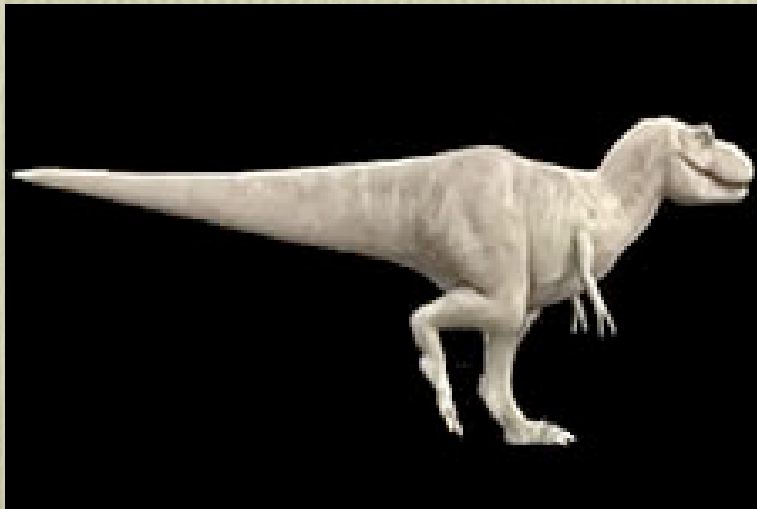
computer model of
vibrating drum head

modes of vibration for a
cylindrical air column

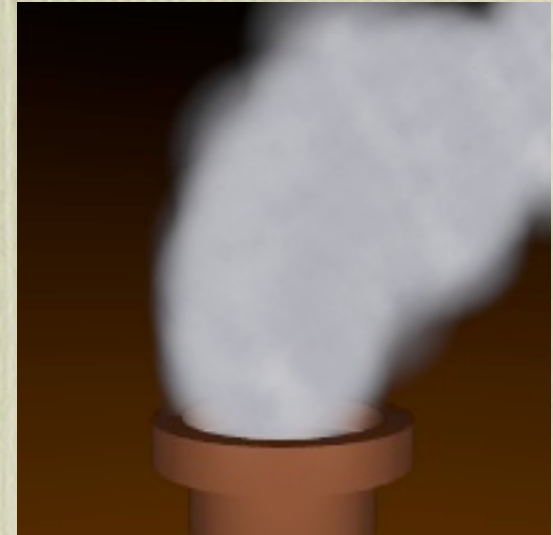


What IS Physical Modelling?

- analogous to computer graphics modelling of physical structures to simulate realistic dynamical behaviour



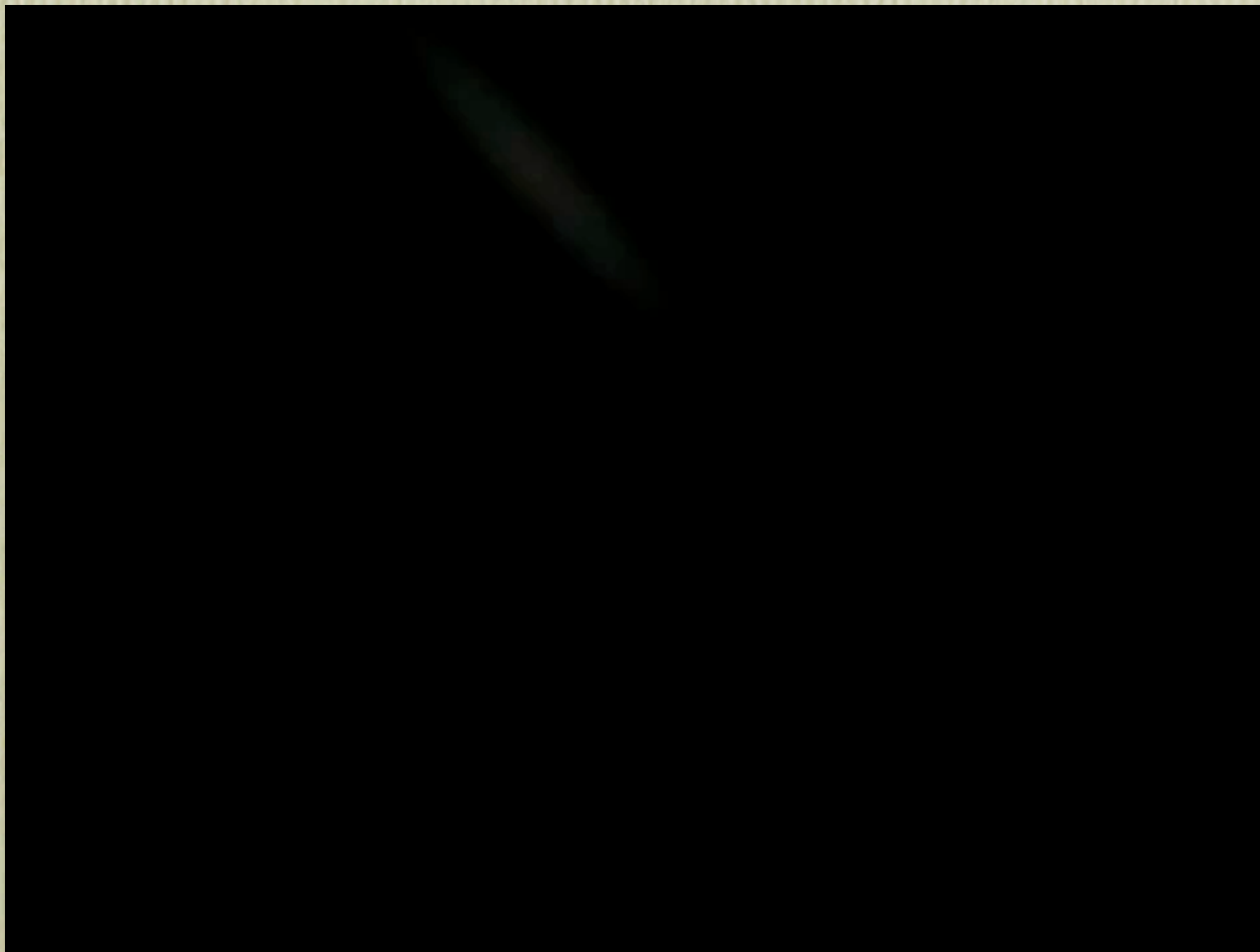
animated dinosaur moves realistically by careful simulation of physical structure



smoke synthesized with physical modelling (particle system)

What IS Physical Modelling?

- colliding galaxies synthesized by John Dubinski, U. of T.
Dept. of Astronomy & Astrophysics



colliding galaxies

What IS Physical Modelling?

- simulates instrumental dynamics (i.e. behaviour)

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Violin physical model, staccato

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Violin physical model, bowing overpressure

The Problem with Physical Modelling...

A More Complete Derivation of the String Wave Equation

Consider an elastic string under tension which is at rest along the x dimension. Let \mathbf{i} , \mathbf{j} , and \mathbf{k} denote the unit vectors in the x , y , and z directions, respectively. When a wave is present, a point $\mathbf{p} = (x, 0, 0)$ originally at x along the string is displaced to some point $\mathbf{a} = \mathbf{p} + d\mathbf{p}$ specified by the displacement vector

$$d\mathbf{p} = \mathbf{i}\xi + \mathbf{j}\eta + \mathbf{k}\zeta.$$

Note that typical derivations of the [wave equation](#) consider only the displacement η in the y direction. This more general treatment is adapted from [118].

The displacement of a neighboring point originally at $\mathbf{q} = (x + dx, 0, 0)$ along the string can be specified as

$$d\mathbf{q} = \mathbf{i}(\xi + d\xi) + \mathbf{j}(\eta + d\eta) + \mathbf{k}(\zeta + d\zeta).$$

Let K denote string tension along x when the string is at rest, and \mathbf{K} denote the vector tension at the point \mathbf{p} in the present displaced scenario under analysis.

The net vector force acting on the infinitesimal string element between points \mathbf{p} and \mathbf{q} is given by the vector sum of the force $-\mathbf{K}$ at \mathbf{p} and the force $\mathbf{K} + (\partial\mathbf{K}/\partial x)dx$ at \mathbf{q} , that is, $(\partial\mathbf{K}/\partial x)dx$. If the string has stiffness, the two forces will in general not be tangent to the string at these points. The

mass of the infinitesimal string element is ϵdx , where ϵ denotes the mass per unit length of the string at rest. Applying Newton's second law gives

$$\frac{\partial\mathbf{K}}{\partial x} = \epsilon \frac{\partial^2\mathbf{p}}{\partial t^2} \quad (\text{F.1})$$

where dx has been canceled on both sides of the equation. Note that no approximations have been made so far.

The next step is to express the force \mathbf{K} in terms of the tension K of the string at rest, the elastic constant of the string, and geometrical factors. The displaced string element \mathbf{pq} is the vector

$$d\mathbf{s} = \mathbf{i}(dx + d\xi) + \mathbf{j}d\eta + \mathbf{k}d\zeta \quad (\text{F.2})$$

$$= \left[\mathbf{i} \left(1 + \frac{\partial\xi}{\partial x} \right) + \mathbf{j} \frac{\partial\eta}{\partial x} + \mathbf{k} \frac{\partial\zeta}{\partial x} \right] dx \quad (\text{F.3})$$

having magnitude

$$ds = \sqrt{\left(1 + \frac{\partial\xi}{\partial x} \right)^2 + \left(\frac{\partial\eta}{\partial x} \right)^2 + \left(\frac{\partial\zeta}{\partial x} \right)^2} dx. \quad (\text{F.4})$$

from Julius O. Smith,
"Physical Audio Signal
Processing" 2006
[http://
ccrma.stanford.edu/](http://ccrma.stanford.edu/)

Karplus-Strong Synthesis

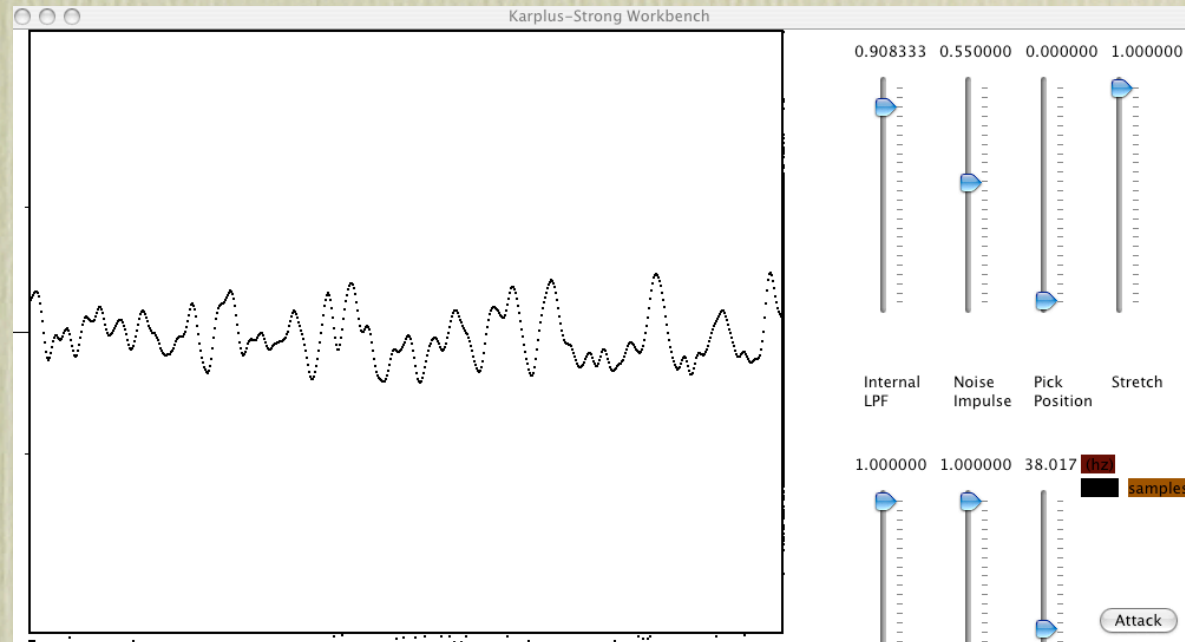
- named after Stanford grad. students Kevin Karplus and Alex Strong
 - Kevin Karplus, Alex Strong (1983). "Digital Synthesis of Plucked String and Drum Timbres". *Computer Music Journal* 7 (2): 43-55.
- efficient method for 8-bit microprocessor
- fill wavetable with random numbers
- average successive samples each time through the loop
 - averaging amounts to:
 - low-pass filtering (frequency domain description)
 - waveform smoothing (time domain description)

MidiForth Karplus Workbench

Steady State Random Noise

Internal Low Pass Filter (LPF)

Varying Pitch



Karplus-Strong Synthesis

- Bruno Degazio - *HeatNoise* (1987)

HeatNoise is a fantasy on the inter-relationship of signal and noise, meaning and error, chaos and order. Noise - taken broadly and metaphorically as the absence of meaning - and the emergence of meaning from noise is presented with sounds synthesized by means of the same fractal process used to generate the structure of the work; with the noisy sounds of speech, the sibilants, plosives and fricatives without which language would be unintelligible; with radio transmissions, including Neil Armstrong's famous non sequitur at the first moon landing; and with sounds, musical and otherwise, that employ noise in various ways to communicate a message.

Out of the opening chaos through the progressively greater disturbances of the underlying order, noise overwhelms meaning until we arrive at the place where the lost messages end - the radio transmissions that were never received, the cries for help that were never heard, the final gasps of those who died alone... Curiously enough, just as researchers in information theory found that indelicate four letter words were the first to emerge from the chaos of random letter orderings, so here we discover that the last sound to be heard as the chaos engulfs us is not profanity but... rock music.

HeatNoise is one of a series of algorithmic compositions applying principles of fractal geometry to music. The structural foundation for the work is an extended rhythmic figure generated by the fractal equation used to describe errors due to thermal noise encountered in data transmission.

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Waveguide Synthesis

- Stanford Prof. Julius O. Smith realized that this looped wavetable was equivalent to a digital representation of a vibrating string (or air column).
- developed the theoretical basis for what later became Waveguide Synthesis
- patented by Stanford in 1989 and licensed to Yamaha in 1994
- Yamaha's previous licensing relationship with Stanford included *FM Synthesis*, which resulted in the best selling synthesizer of the period, (Yamaha DX7) and the 2nd most lucrative licensing agreement in Stanford's history (\$20,000,000)

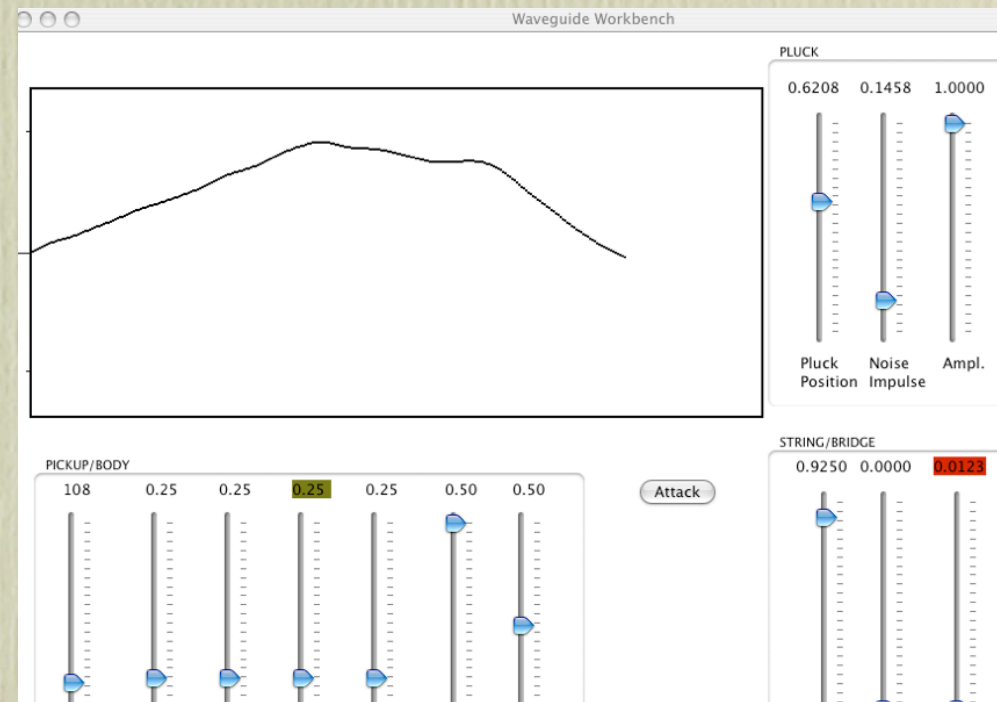
MidiForth Waveguide Workbench

Pluck Position

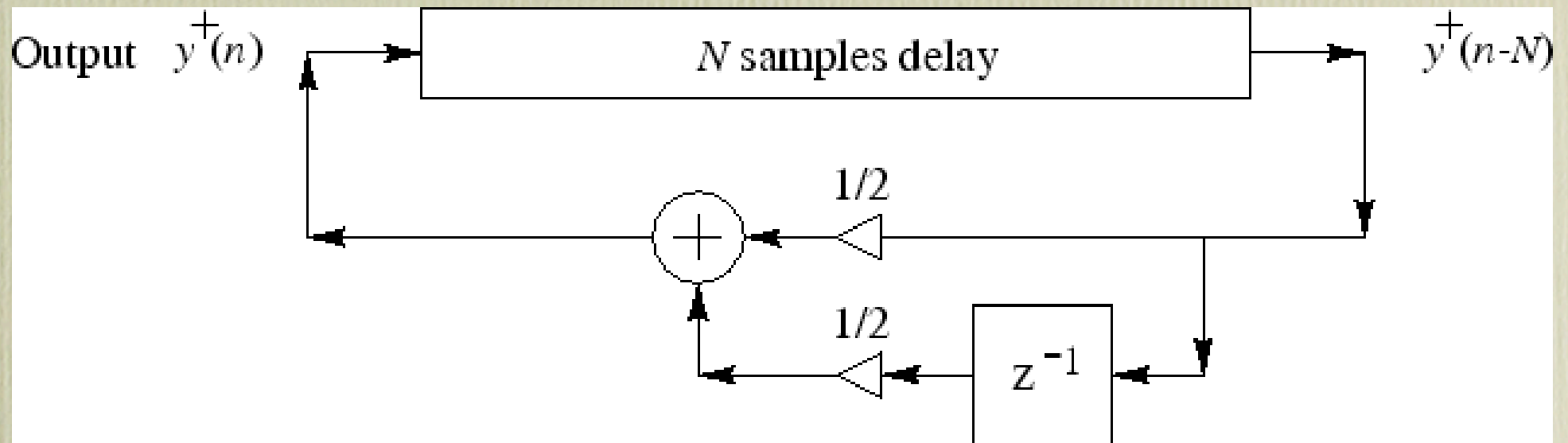
Bridge LPF

Varying Pickup Position

Varying Pitch



Karplus-Strong Synthesis



JOS Proposed Clarinet Model (1986)

1.1. The Clarinet

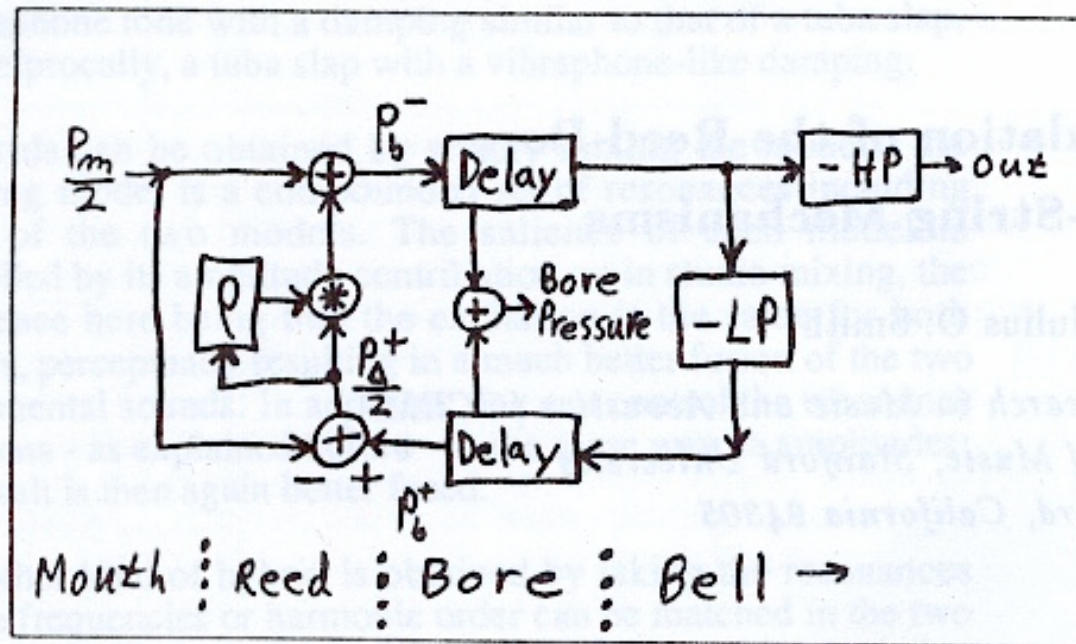
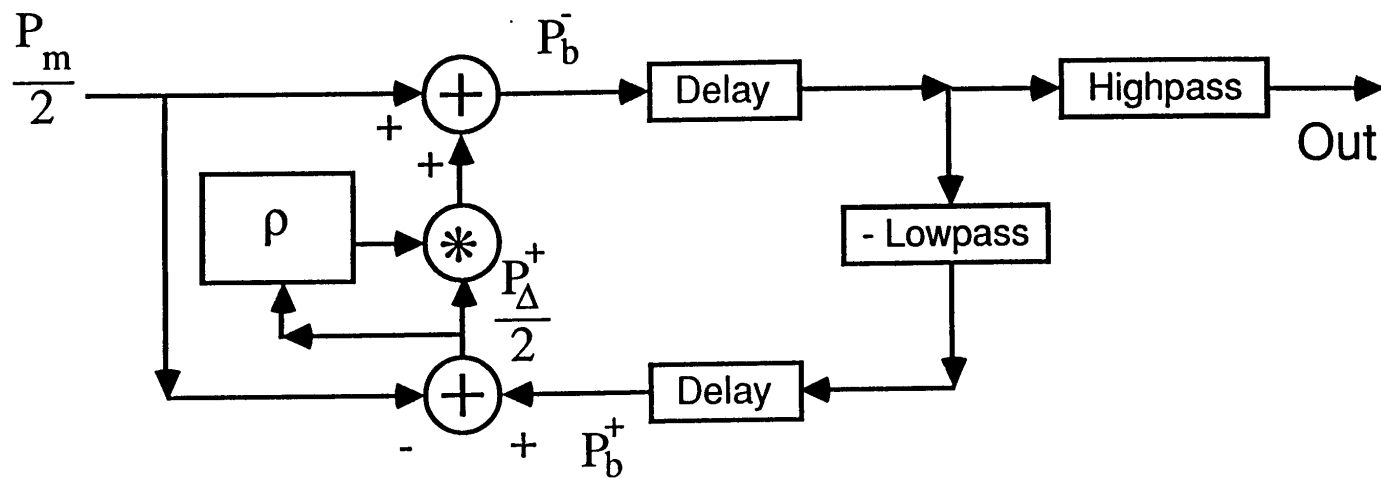


Figure 1. Model of a single-reed, cylindrical-bore woodwind.

from "Efficient Simulation of the Reed-Bore and Bow-String Mechanisms", Proceedings of the International Computer Music Conference, The Hague, 1986

JOS Proposed Clarinet Model (1991)

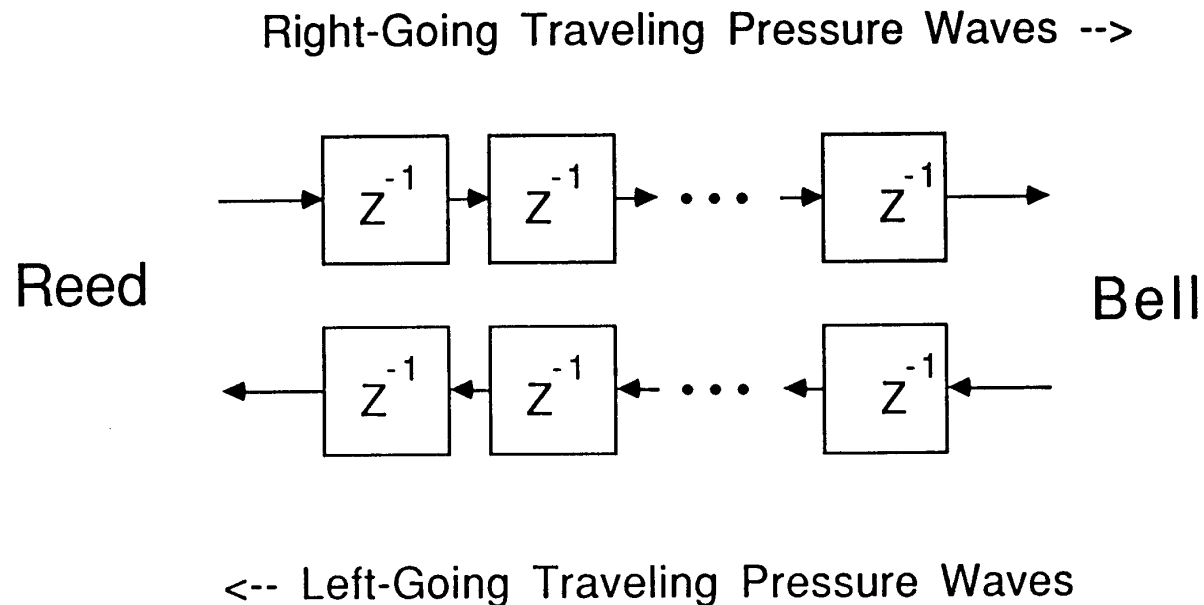
Proposed Clarinet Implementation



$$P_b^- = \rho \left(\frac{P_\Delta^+}{2} \right) \frac{P_\Delta^+}{2} + \frac{P_m}{2}$$

JOS Proposed Clarinet Model (1991)

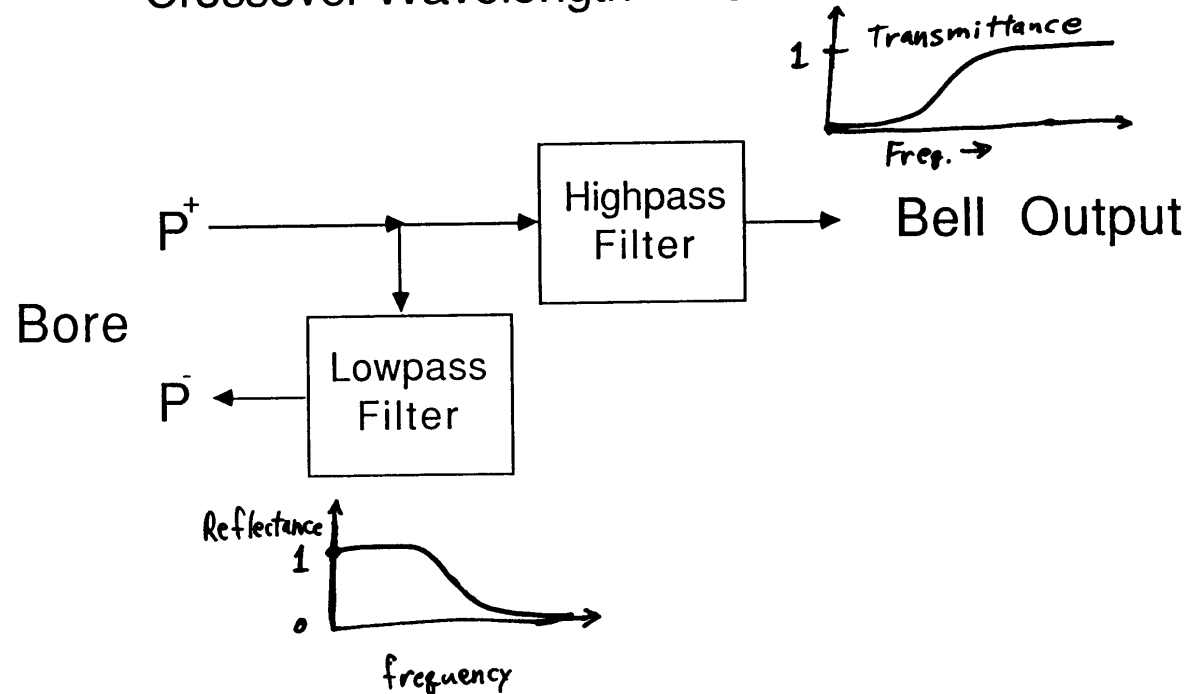
Clarinet Bore = Digital Waveguide
(Bi-Directional Delay Line)



JOS Proposed Clarinet Model (1991)

Bell = Crossover Network

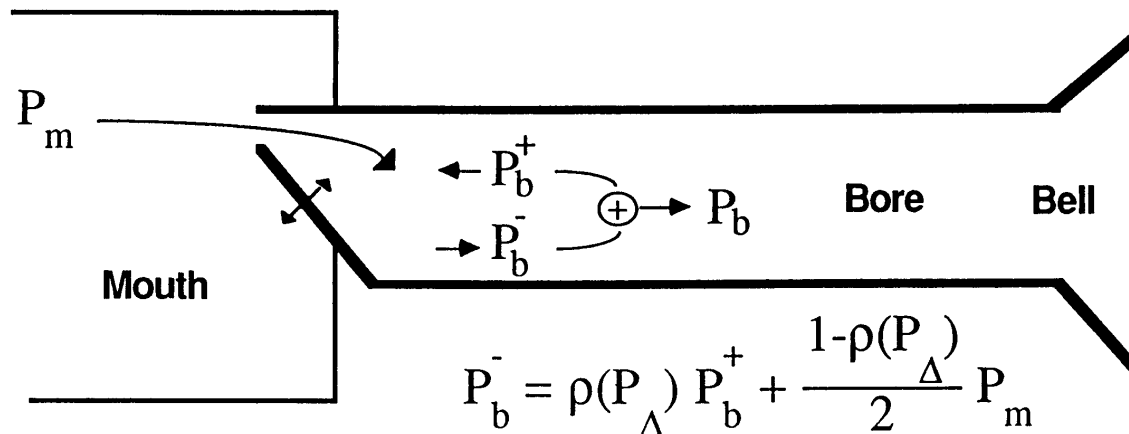
Crossover Wavelength = Bore Diameter



JOS Proposed Clarinet Model (1991)

Reed = Pressure-Controlled Valve

Bore sees a time-varying reflection-coefficient plus a time-varying mouth-pressure input



$$P_b^- = \rho(P_\Delta) P_b^+ + \frac{1 - \rho(P_\Delta)}{2} P_m$$

$$P_\Delta = P_b - P_m$$

$$P_b = P_b^+ + P_b^-$$

JOS Proposed Violin Model (1986)

1.2. The Bowed String

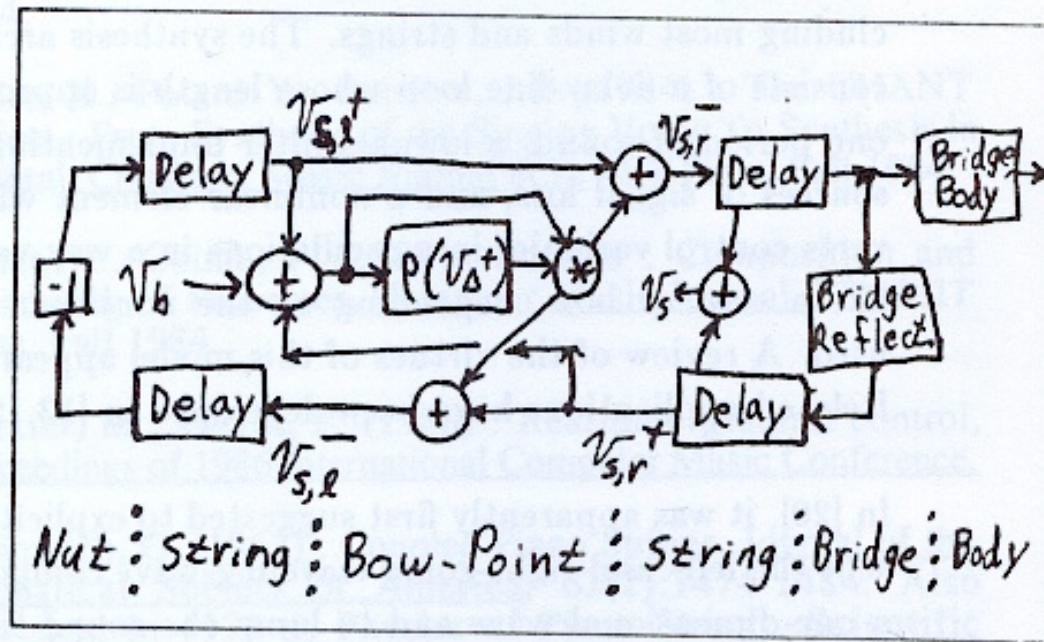


Figure 2. Basic model for a bowed string.

from "Efficient Simulation of the Reed-Bore and Bow-String Mechanisms", Proceedings of the International Computer Music Conference, The Hague, 1986

Yamaha VLI Synthesizer

- Julius Smith & Stanford U. patented these techniques in late 1980s
- Yamaha licensed the patent in early '90s
- VLI synthesizer was the first product of Yamaha's license of Stanford's waveguide technology
- 2 voices, 48 khz, 16 bit
- optimized for simulation of woodwind and brass instruments, esp. saxophones

VLI Architecture I



	<i>Driver</i>	<i>Resonator</i>	<i>Modifier</i>
<i>WW</i>	reed	Pipe	Bell
<i>Brass</i>	lips	Pipe	Bell
<i>String</i>	bow	String	Body

VLI Pipe Parameters

PIPE/STRING

Straight Horn Insertion

Straight Horn1 Length 0.021ms

Straight 2/Conical Len 0.021ms

Short Length Mode:

High Freq Abs. Mode

Damping/Decay

Register Key Open D 5

Delay Mode:

Ratio 1 to 2 1.000

Ratio 2 to Conical 1.000

Short Length/Ratio

Absorption 15000 h

1st Harmonic Dampening
(Low/High Balance)

VLI - Tube Shape



FIGURE 2: Absorption High. Cone tapered, minimum flare.

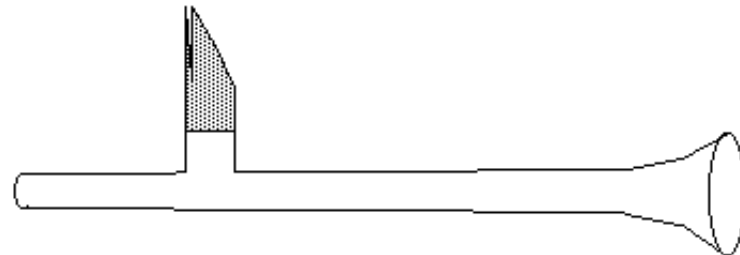


FIGURE 3: Absorption Low. Cone nearly straight, greater flaring.

Effect of Absorption on shape
of Tube & Bell

Imitative Synthesis I - Clarinet

prg. 033 ClasClarBD

overblowing

multiphonics

Imitative Synthesis I - Clarinet

Mozart – Clarinet Quintet

prg. 033 ClasClarBD

Allegro. W. A. MOZART (1756-1791)

Clarinetto in A.

The musical score is written for Clarinet in A. It begins with a treble clef and a common time signature. The tempo is marked 'Allegro.' and the composer is 'W. A. MOZART (1756-1791)'. The score is divided into four systems of music. The first system starts with a whole rest, followed by a measure with a finger number '5' above it, and then a series of eighth and sixteenth notes. A dynamic marking 'p' is placed below the first note of the second measure. The second system starts with a measure containing a finger number '5' above it, followed by more eighth and sixteenth notes. The third system continues with eighth and sixteenth notes. The fourth system starts with a measure containing a finger number '8' above it, followed by eighth and sixteenth notes. The score ends with a final note and a fermata.

Imitative Synthesis I - Clarinet

Mozart – Clarinet Quintet

prg. 033 ClasClarBD

Allegro. W. A. MOZART (1756-1791)

Clarinetto in A.

The image displays a musical score for the Clarinet part of Mozart's Clarinet Quintet. It features four staves of music in treble clef with a common time signature (C). The tempo is marked 'Allegro.' and the composer is identified as 'W. A. MOZART (1756-1791)'. The instrument is specified as 'Clarinetto in A.'. The score includes dynamic markings such as 'p' (piano) and fingering numbers like '5' and '8'. The music consists of various rhythmic patterns, including eighth and sixteenth notes, and rests.

Imitative Synthesis 1 - Flute

Claude Debussy - Syrinx

Imitative Synthesis 1 - Flute

Claude Debussy - Syrinx

Imitative Synthesis 2 - Oboe

prg. 034 Oboe2md

- J.S. Bach BWV 1060 - Double Concerto

-all instruments are physically modelled except harpsichord

Imitative Synthesis 2 - Oboe

prg. 034 Oboe2md

- J.S. Bach BWV 1060 - Double Concerto

Imitative Synthesis 2 - Saxophone

prg 035 - Desmond

Ornithology

By Charlie Parker and Benny Harris

'BIRD SYMBOLS'
C. PARKER 407

$\text{♩} = 236$

1 **DRUMS** 3

2 A7 D D- G7

3 C7 F#6 B7 1 E- 3 B7+9 G#-

The image shows a handwritten musical score for saxophone. It consists of three staves of music. The first staff is marked with a tempo of quarter note = 236 and a 'DRUMS' section with a 3-measure rest. The second staff contains complex rhythmic patterns with chord markings A7, D, D-, and G7. The third staff continues the complex patterns with chord markings C7, F#6, B7, E-, B7+9, and G#-. The notation includes many beamed notes and rests, characteristic of Charlie Parker's style.

Imitative Synthesis 2 - French Horn


prg 037

**Instrumental Behavior:
The Harmonic Series**

Imitative Synthesis 2 - French Horn

Strauss – Till Eulenspiegel, op. 28

etwas gemächlicher 7 28 *III. Horn. zart*



prg 038

6 *atmählich lebhafter*



Volles Zeitmass. (sehr lebhaft) 6 29 4



fp *mf* *fp* *f*



1 30 *f* *mf* *f*



2 1 *mf cresc.* *ff* *f*



31 *ff* *ff* *ff molto marcato*



ff



Imitative Synthesis 1 - Woodwind Quintet

- Malcolm Arnold - Sea Shanty #1, arranged for
“Virtual” Woodwind Quintet

Imitative Synthesis 1 - Woodwind Quintet

- Malcolm Arnold - Sea Shanty #1, arranged for “Virtual” Woodwind Quintet

Non-Imitative Synthesis

- o40 - Mode X
- o41 - DblSqwaker
- o42- Mad Tube
- o43 - Waterphone

Conclusion

Bruno Degazio: *Algorithmic Animal Five* (2003)

- all instruments (except piano) are physically modelled
- musical structure is algorithmically generated on Schenkerian principles using my *Transformation Engine* composition software

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Bruno Degazio: *Algorithmic Animal Five* (2003) (3:50)

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- musical structure is algorithmically generated on Schenkerian principles using my *Transformation Engine* composition software

Why Bother?

- **Elegance** - sounds generated from first principles rather than through *ad hoc* accumulation of imitative features
- **Acoustically Detailed**
 - onset transients & note transitions
- **Synthesis Parameters have real-world counterparts**
 - can be extended beyond real-world limits
- **Playability**
 - responsive
 - expressive
 - realistic
 - unpredictable