Virtual Acoustic Musical Instruments: Review of Models and Selected Research

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Oct. 19, 2005

Presentation Overheads
IEEE Workshop on Applications of Signal Processing
to Audio and Acoustics (WASPAA-05)
Oct. 16–19, 2005

http://ccrma.stanford.edu/~jos/Mohonk05/

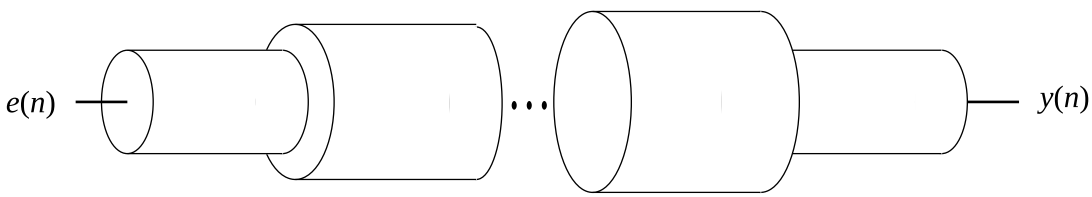
Outline

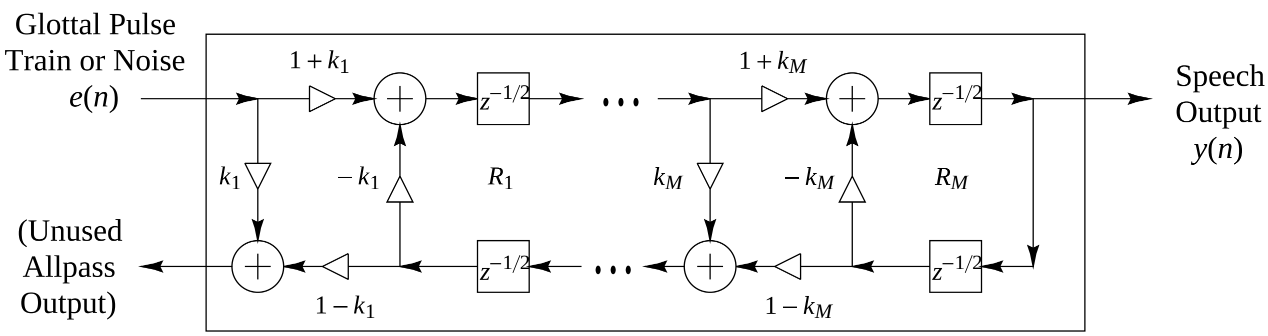
In *historical order*, with selected updates:

- Voice Synthesis
- Karplus-Strong Algorithm
- Waveguide Synthesis
- Commuted Synthesis
- Virtual Analog

Voice Modeling

Linear Prediction (LP) Vocal Tract Model





Kelly-Lochbaum Vocal Tract Model (Piecewise Cylindrical)

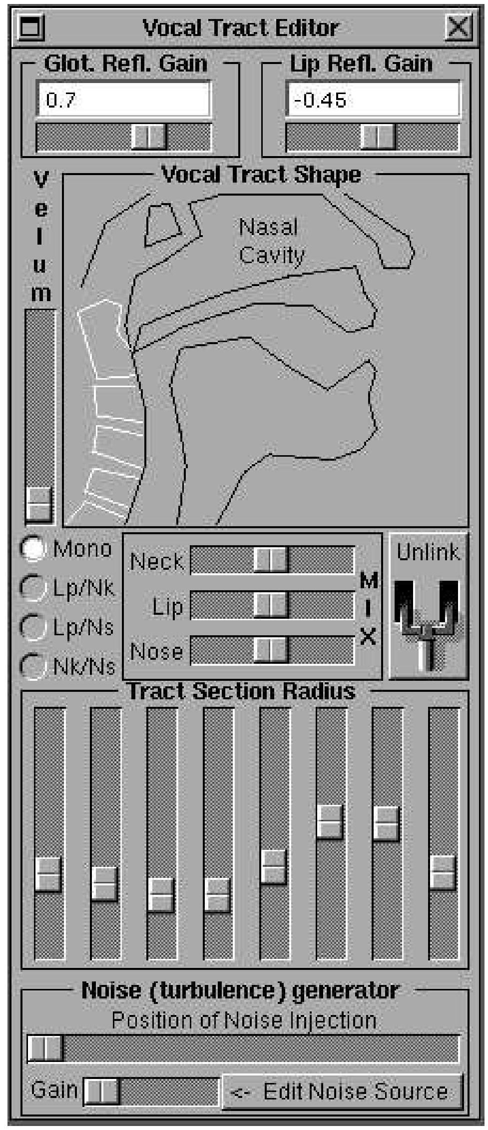
John L. Kelly and Carol Lochbaum (1962)

Sound Example

"Bicycle Built for Two": (WAV) (MP3)

- Vocal part by Kelly and Lochbaum (1961)
- Musical accompaniment by Max Mathews
- Computed on an IBM 704
- Based on Russian speech-vowel data from Gunnar Fant's recent book
- Probably the first digital physical-modeling synthesis sound example by any method
- Inspired Arthur C. Clarke to adapt it for "2001: A Space Odyssey" — the computer's "first song"

“Shiela” Sound Examples by Perry Cook (1990)



- Diphones: (WAV) (MP3)
- Nasals: (WAV) (MP3)
- Scales: (WAV) (MP3)
- “Shiela”: (WAV) (MP3)

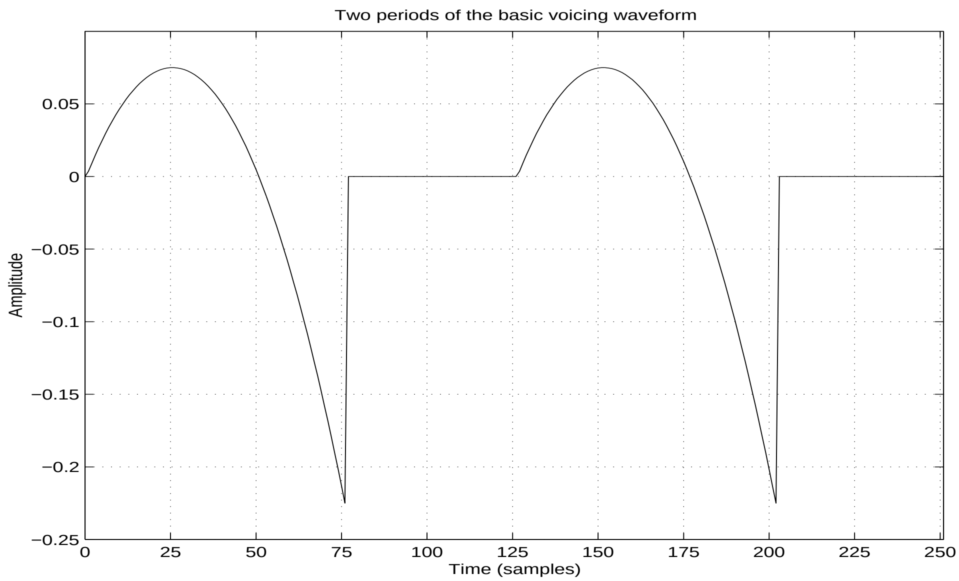
Recent Voice Modeling Efforts

Linear Prediction (LP) Vocal Tract Model

- Can be interpreted as a modified Kelly-Lochbaum model
- In linear prediction, the glottal excitation must be an
  - *impulse*, or
  - *white noise*

  This prevents LP from finding a physical vocal-tract model
- A *more realistic glottal waveform* $e(n)$ is needed before the vocal tract filter can have the "right shape"
- How to augment LPC in this direction without going to a full-blown *articulatory synthesis model?*
  - Jointly estimate *glottal waveform* $e(n)$ so vocal-tract filter can have the "right shape"

Klatt Derivative Glottal Wave



- Good for estimation:
  - Truncated *parabola* each period
  - Coefficients easily fit to *phase-aligned* inverse-filter output

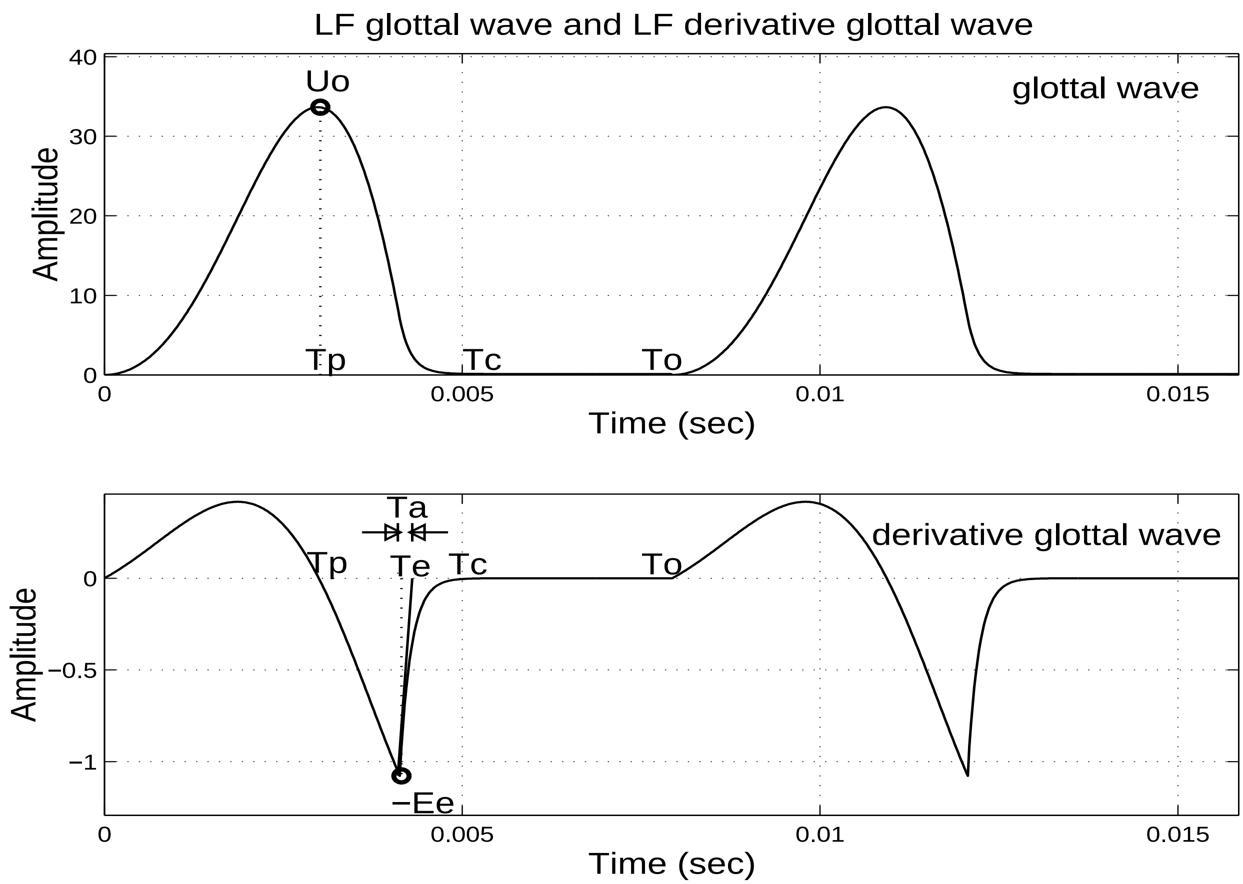
Sequential Unconstrained Minimization

(Hui Ling Lu, 2002)

Klatt glottal (parabola) parameters are estimated *jointly* with vocal tract filter coefficients

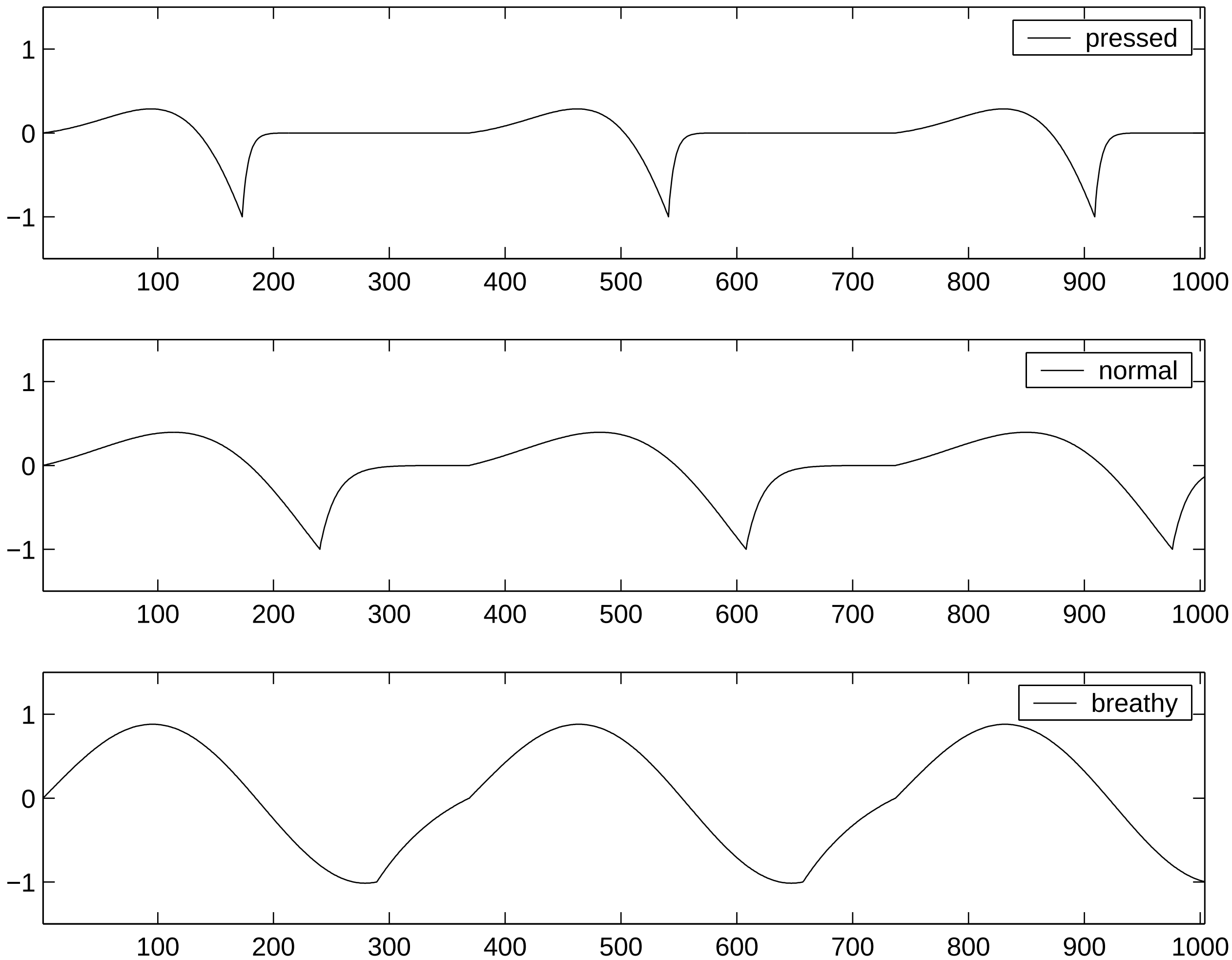
- Formulation resembles that of the *equation error method for system identification*
- For *phase alignement*, we estimate
  - pitch (time varying)
  - glottal closure instant each period
- Optimization is *convex* in all but the phase-alignment dimension

Liljencrantz-Fant Derivative Glottal Wave Model



- Better for *intuitively parametrized expressive synthesis*
- LF model parameters are fit to *inverse filter* output
- Use of Klatt model in forming filter estimate yields a "more physical" filter than LP

Parametrized Phonation Types



Sound Examples by Hui Ling Lu

- Original: (WAV) (MP3)
- Synthesized:
  - Pressed Phonation: (WAV) (MP3)
  - Normal Phonation: (WAV) (MP3)
  - Breathy Phonation: (WAV) (MP3)
- Original: (WAV) (MP3)
- Synthesis 1: (WAV) (MP3)
- Synthesis 2: (WAV) (MP3)

where

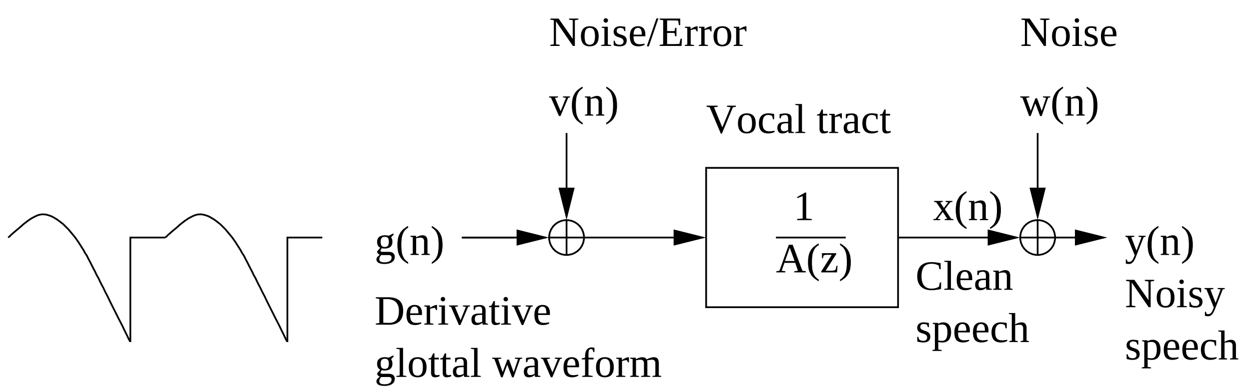
- Synthesis 1 = Estimated Vocal Tract driven by estimated KLGLOT88 Derivative Glottal Wave (Pressed)
- Synthesis 2 = Estimated Vocal Tract driven by the fitted LF Derivative Glottal Wave (Pressed)

`Google` search: *singing synthesizer vocal texture control* (Hui Ling Lu's thesis page at CCRMA)

Voice Model Estimation

(Pamornpol Jinachitra)

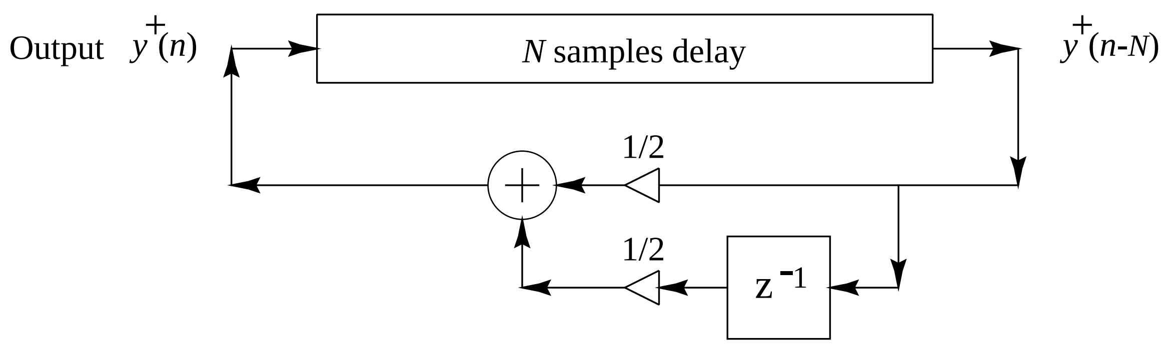
(to be presented later this morning)



System Diagram

- Parametric source-filter model of voice + noise
- State-space framework with derivative glottal waveform as input and A model for dynamics
- Jointly estimate AR parameters and glottal source parameters using EM algorithm with Kalman smoothing
- Reconstruct a clean voice using Kelly-Lochbaum and estimated parameters

Karplus-Strong Algorithm



- Discovered (1978) as a self-modifying wavetable synthesis algorithm
- "Vintage" 8-bit sound examples:
  - Original Plucked String: (AIFF) (MP3)
  - Drum: (AIFF) (MP3)
  - Stretched Drum: (AIFF) (MP3)
- STK Plucked String: (WAV) (MP3)
  - Plucked String 1: (WAV) (MP3)
  - Plucked String 2: (WAV) (MP3)
  - Plucked String 3: (WAV) (MP3)
  - Plucked String 4: (WAV) (MP3)

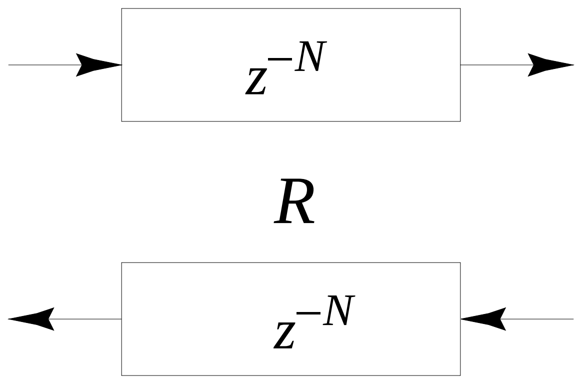
Interpretations of the Karplus-Strong Algorithm

The Karplus-Strong structure can be *interpreted* as a

- *pitch prediction filter* from the Codebook-Excited Linear Prediction (CELP) standard (*periodic LPC* synthesis)
- *feedback comb filter* with *lowpassed feedback* used earlier by James A. Moorer for recursively modeling *wall-to-wall echoes* ("About This Reverberation Business")
- simplified digital waveguide model

Digital Waveguide Models

A *lossless digital waveguide* $\stackrel{\Delta}{=}$ *bidirectional delay line* at some *wave impedance* R:



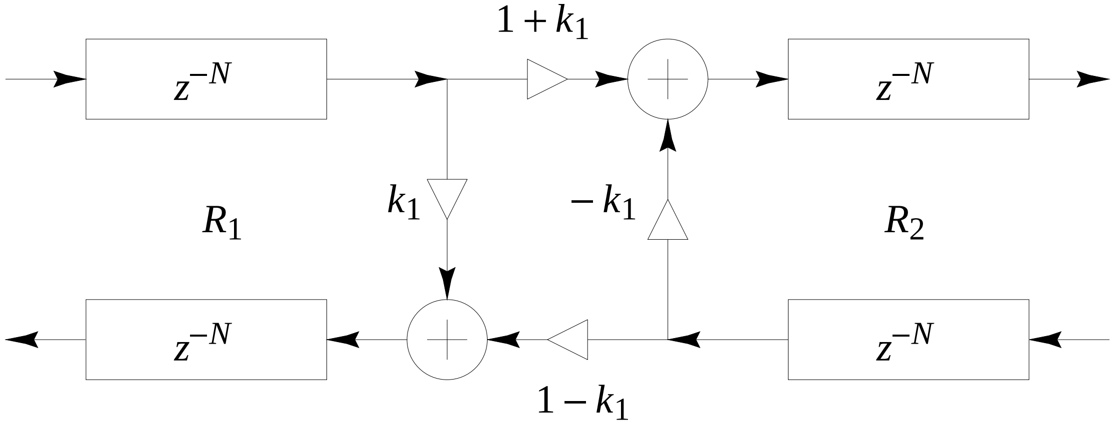
Useful for efficient models of

- strings
- bores
- plane waves
- conical waves

Signal Scattering

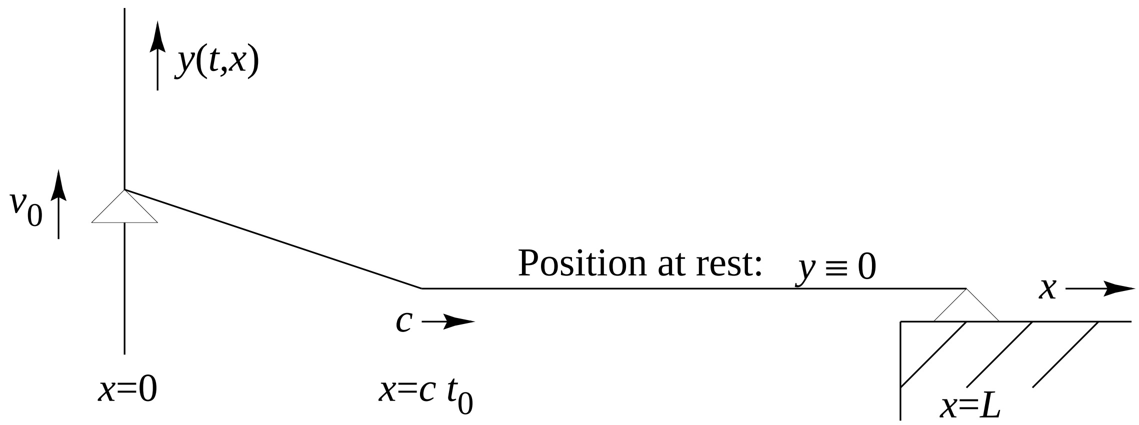
Signal *scattering* is caused by a *change* in wave impedance R:

$$k_1 = \frac{R_2 - R_1}{R_2 + R_1}$$



If the wave impedance changes *every sample*, the Kelly-Lochbaum vocal-tract model results.

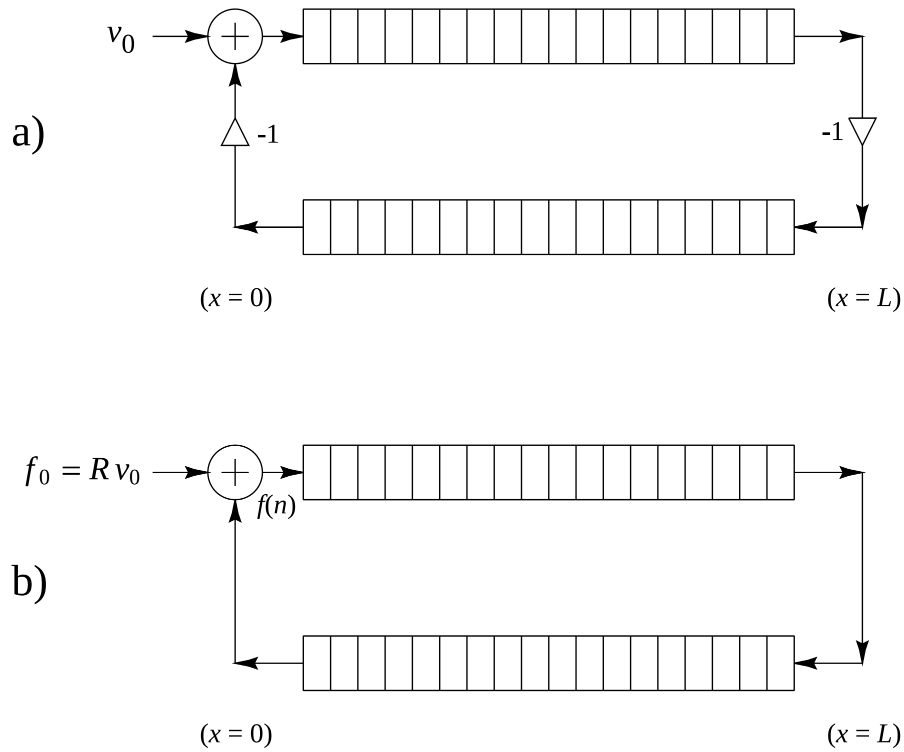
Moving Termination: Ideal String



Moving rigid termination for an ideal string.

- Left endpoint moved at velocity v_0
- External force $f_0 = Rv_0$
- $R = \sqrt{K\epsilon}$ is the *wave impedance* (for transverse waves)
- Relevant to *bowed strings* (when bow pulls string)
- String moves with speed v_0 or 0 only
- String is always one or two straight segments
- "Helmholtz corner" (slope discontinuity) shuttles back and forth at speed $c = \sqrt{K/\epsilon}$

Digital Waveguide "Equivalent Circuits"

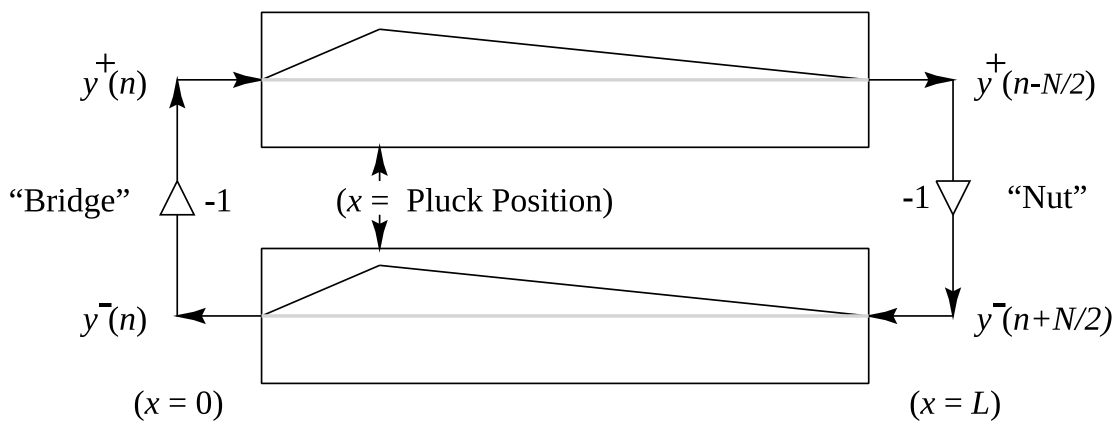


a) Velocity waves.
b) Force waves.

Animation:

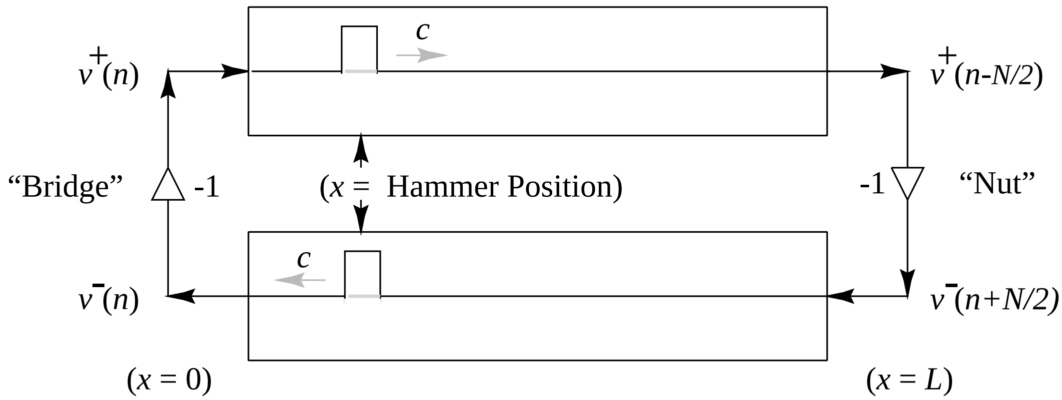
`http://ccrma.stanford.edu/~jos/swgt/movet.html`

Ideal Plucked String (Displacement Waves)



- Load each delay line with *half* of initial string displacement
- Sum of upper and lower delay lines = string displacement

Ideal Struck String (Velocity Waves)

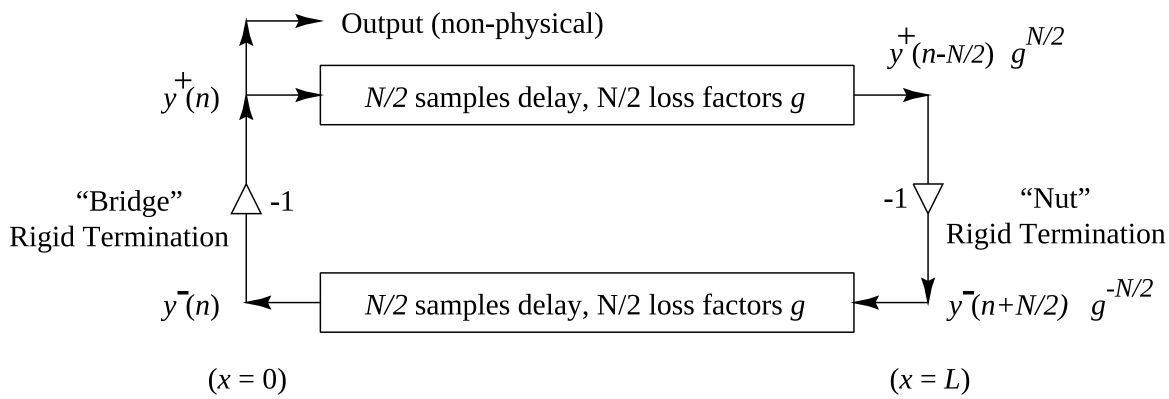


Hammer strike = *momentum transfer* = velocity step:

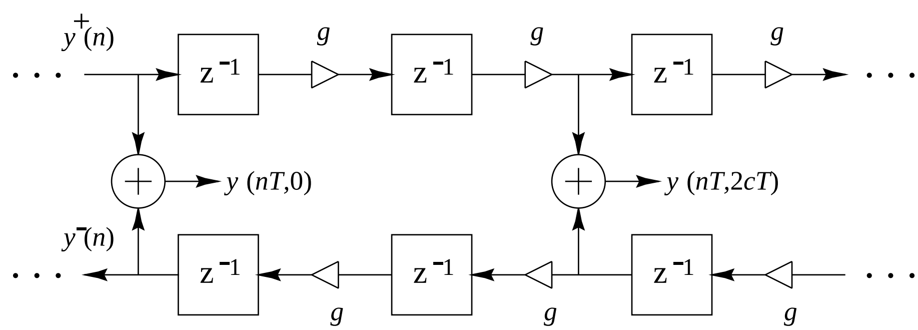
$$m_h v_h(0-) = (m_h + m_s) v_s(0+)$$

Digital Waveguide Interpretation of the Karplus-Strong Algorithm

Begin with an ideal *damped* string model:

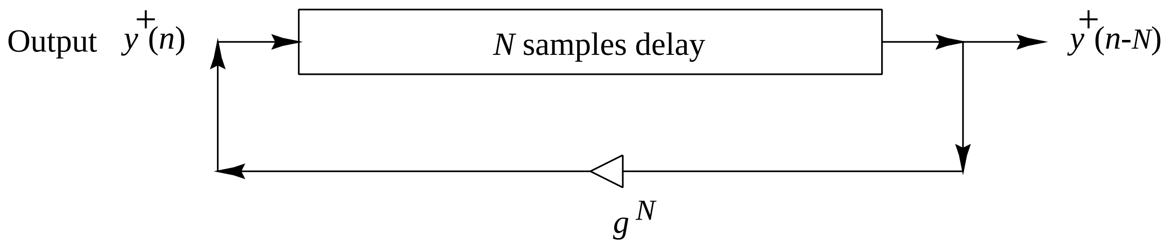


- Rigidly terminated string with distributed "resistive" losses (force $\propto$ velocity)
- Sampled wave-equation solution yields N loss factors g embedded between the delay-line elements:



- Note that loss factors g *commute* with delay elements

Equivalent System: Gain Elements Commuted



All N loss factors g have been "pushed" through delay elements and combined at a *single* point.

Computational Savings

- $f_s = 50$kHz, $f_1 = 100 Hz \Rightarrow$ delay $= 500$
- Multiplies reduced by two orders of magnitude
- Input-output transfer function unchanged
- Round-off errors reduced

Frequency-Dependent Damping

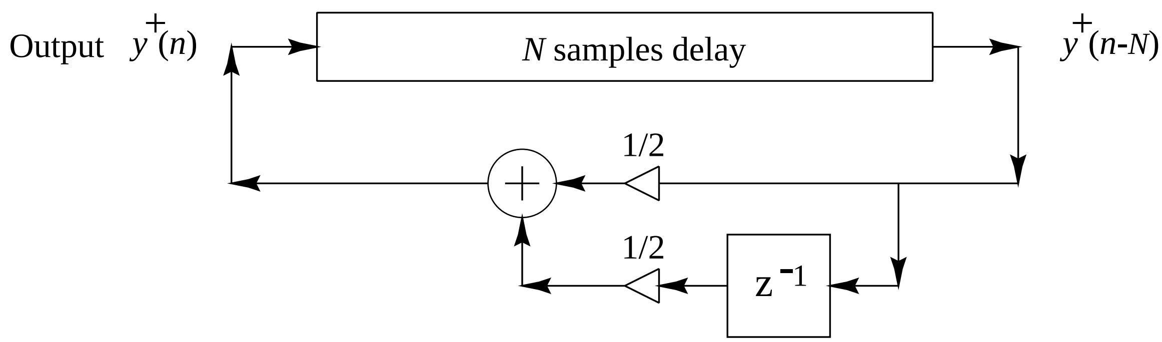
- Loss factors g should really be digital filters
- Gains in nature typically decrease with frequency
- Loop gain may not exceed 1 (for stability)
- Such filters *also commute* with delay elements (LTI)
- Typically only *one* gain filter used per loop

Simplest Frequency-Dependent Loop Filter

$$\boxed{\hat{G}(z) = b_0 + b_1 z^{-1}}$$

- Uniform delay $\Rightarrow b_0 = b_1$ ($\Rightarrow$ delay $=$ 1/2 sample)
- Zero damping at dc $\Rightarrow b_0 + b_1 = 1$
  $\Rightarrow b_0 = b_1 = 1/2$
  $\Rightarrow$
  $$\boxed{\hat{G}(e^{j\omega T}) = \cos\left(\omega T/2\right), \quad |\omega| \leq \pi f_s}$$
- This is precisely the Karplus-Strong loop filter!

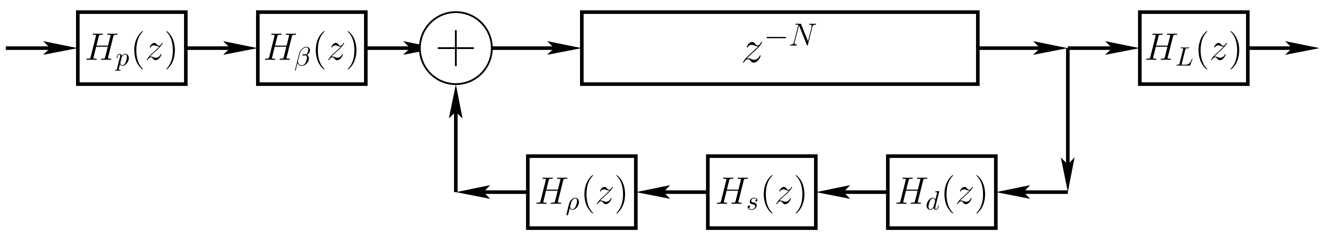
Karplus-Strong Algorithm



Physical Interpretation

- Delay line is initialized with *noise* (random numbers)
- Therefore, assuming a *displacement-wave* simulation:
  - Initial string *displacement* = *sum* of delay-line halves
  - Initial string *velocity* determined by the *difference* of delay-line halves
- The Karplus-Strong "string" is thus *plucked and struck* by random amounts along the *entire length of the string!* (the "splucked string"?)
- Karplus-Strong feedback filter corresponds to the *simplest possible damping filter* for an ideal string

Extended Karplus-Strong (EKS) Algorithm (Jaffe-Smith 1983)



$$N = \text{pitch period } (2\times \text{ string length}) \text{ in samples}$$

$$H_p(z) = \frac{1-p}{1-p\,z^{-1}} = \text{pick-direction lowpass filter}$$

$$H_\beta(z) = 1 - z^{-\beta N} = \text{pick-position comb filter, } \beta \in (0,1)$$

$$H_d(z) = \text{string-damping filter (one/two poles/zeros typical)}$$

$$H_s(z) = \text{string-stiffness allpass filter (several poles and zeros)}$$

$$H_\rho(z) = \frac{\rho(N) - z^{-1}}{1 - \rho(N)\,z^{-1}} = \text{first-order string-tuning allpass filter}$$

$$H_L(z) = \frac{1-R_L}{1-R_L\,z^{-1}} = \text{dynamic-level lowpass filter}$$

EKS Sound Example

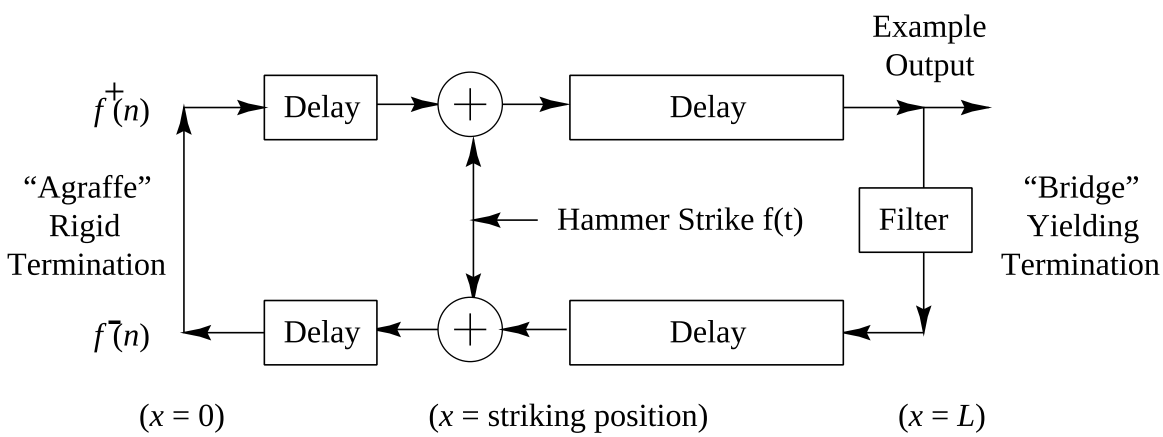
Bach A-Minor Concerto—Orchestra Part: (WAV) (MP3)

- Executes in real time on one Motorola DSP56001 (20 MHz clock, 128K SRAM)
- Developed for the NeXT Computer introduction at Davies Symphony Hall, San Francisco, 1989
- Solo violin part was played live by Dan Kobialka of the San Francisco Symphony

Example EKS Extension

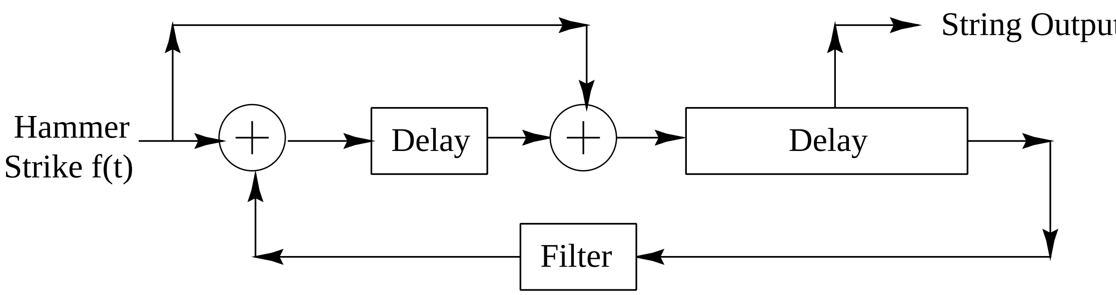
Many of the Karplus-Strong algorithm extensions were based on its *physical interpretation*:

String Excited Externally at One Point



"Waveguide Canonical Form"

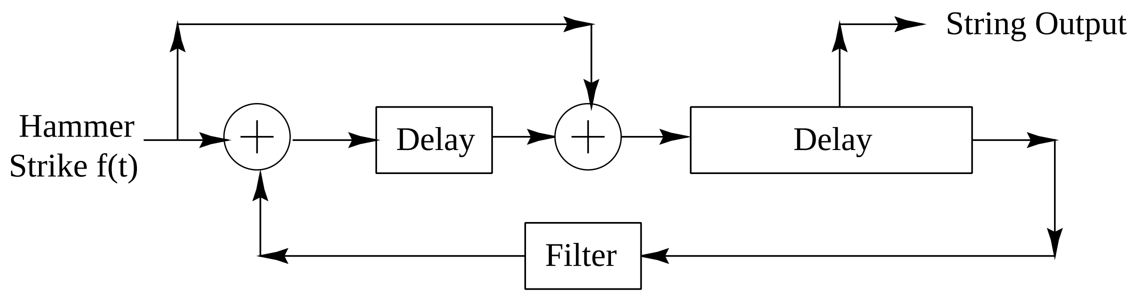
Equivalent System: Delay Consolidation



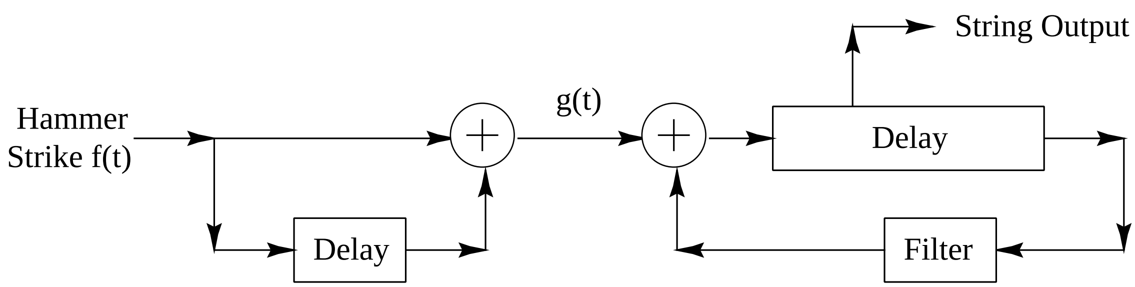
Finally, we "pull out" the comb-filter component:

EKS "Pick Position" Extension

Equivalent System: Delay-Lines Consolidated

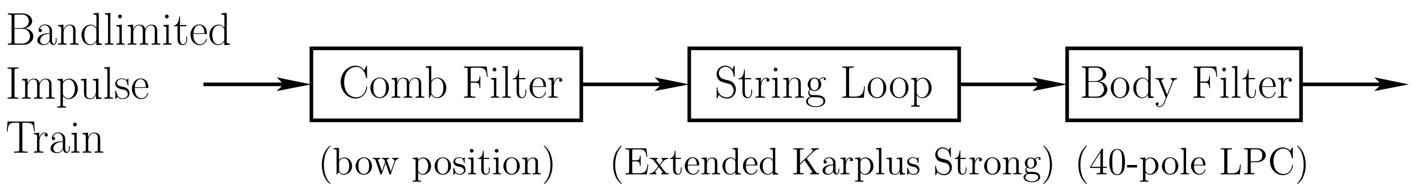


Equivalent System: Comb Filter Factored Out



- *Excitation Position* controlled by left delay-line length
- *Fundamental Frequency* controlled by right delay-line length

PLPC Cello (1982)

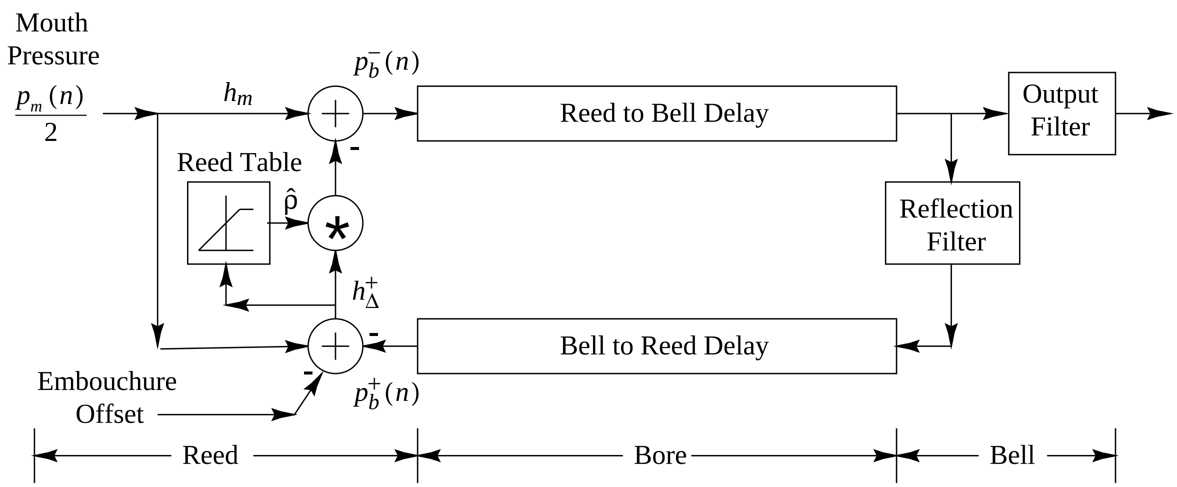


- Periodic LPC used to estimate string-loop filter
- Normal LPC used for body model (40 poles)
- Excitation = Bandlimited impulse train:

$$\sum_{k=1}^{K} \cos(k\omega_0 t) = \frac{\sin[(K+1/2)\omega_0 t]}{2\sin(\omega_0 t/2)} - \frac{1}{2}$$

- Bow-position simulation = variable-delay differencing comb filter (direct from physical interpretation)
- **Sound Example:**
  Moving Bow-Stroke Example: (WAV) (MP3)
  (Bowing point moves toward the "bridge")

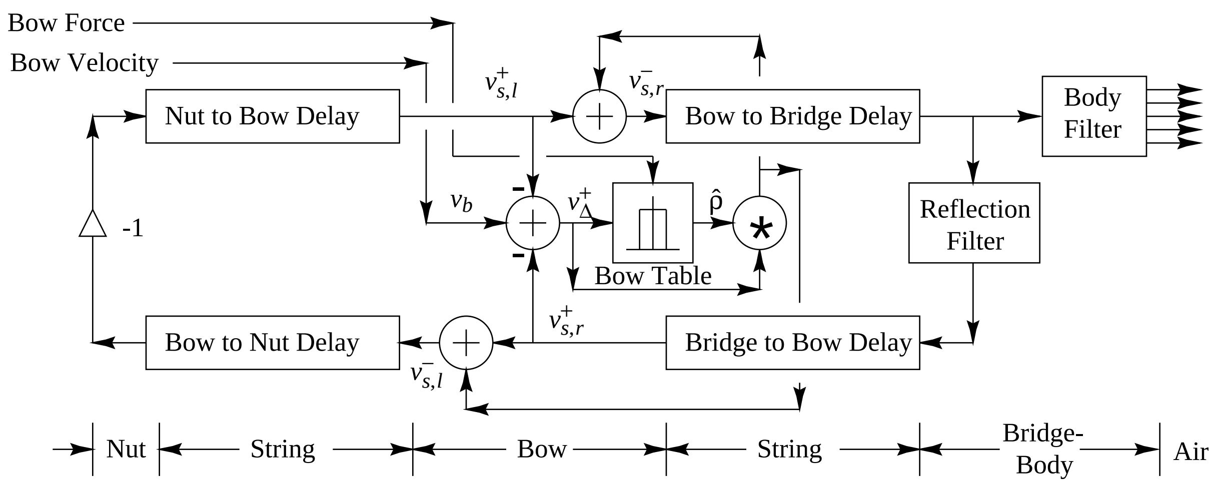
Single-Reed Instruments (1986)



Sound Examples

- STK Clarinet: (WAV) (MP3)
- Staccato Systems Slide Flute (based on STK flute): (WAV) (MP3)
- Yamaha VL1 Shakuhachi: (WAV) (MP3)
- Yamaha VL1 Oboe and Bassoon: (WAV) (MP3)
- VL1 Tenor Saxophone: (WAV) (MP3)
- `Google` search: *STK clarinet*
- Synthesis Tool Kit (STK) by Perry Cook, Gary Scavone, and others, distributed by CCRMA:
  `http://ccrma.stanford.edu/CCRMA/Software/STK/`

Bowed Strings (1986)



- Reflection filter summarizes all losses per period (due to bridge, bow, finger, etc.)
- Bow-string junction = *memoryless* lookup table (or segmented polynomial) ⇒ no thermodynamic model in this version
- Bow-hair dynamics neglected
- Finite bow width neglected

Bowed String Sound Examples

Cell sound examples by Stanford EE graduate student **Peder Larson** using the Synthesis Tool Kit (STK) by Perry Cook and Gary Scavone:

- STK Bowed class, no modifications: (WAV) (MP3)
- Hyperbolic Bow-String Junction, including: (WAV) (MP3)
  - Torsional waves: (WAV) (MP3)
  - Finite Bow Width: (WAV) (MP3)
  - Finite Bow Width and Torsional waves: (WAV) (MP3)
  - Finite Bow Width and Body Filter: (WAV) (MP3)
  - Torsional waves and Body Filter: (WAV) (MP3)
  - Finite Bow Width,Torsional waves, Body Filter: (WAV) (MP3)
  - String Dispersion (Stiffness): (WAV) (MP3)
  - Including Torsional waves and dispersion: (WAV) (MP3)
  - Finite Bow Width and Dispersion: (WAV) (MP3)
  - Finite Bow Width, Torsional Waves, Dispersion: (WAV) (MP3)

- Finite Bow Width, Body Filter, and Dispersion: (WAV) (MP3)
- Torsional waves, Body Filter, and Dispersion: (WAV) (MP3)
- Same plus Finite Bow Width: (WAV) (MP3)

Cello Examples Using All Features

- Stacatto Notes: (WAV) (MP3)
- Bach's First Suite for Unaccompanied Cello: (WAV) (MP3)

Staccato notes created with short strokes of high bow pressure (like a bouncing bow)

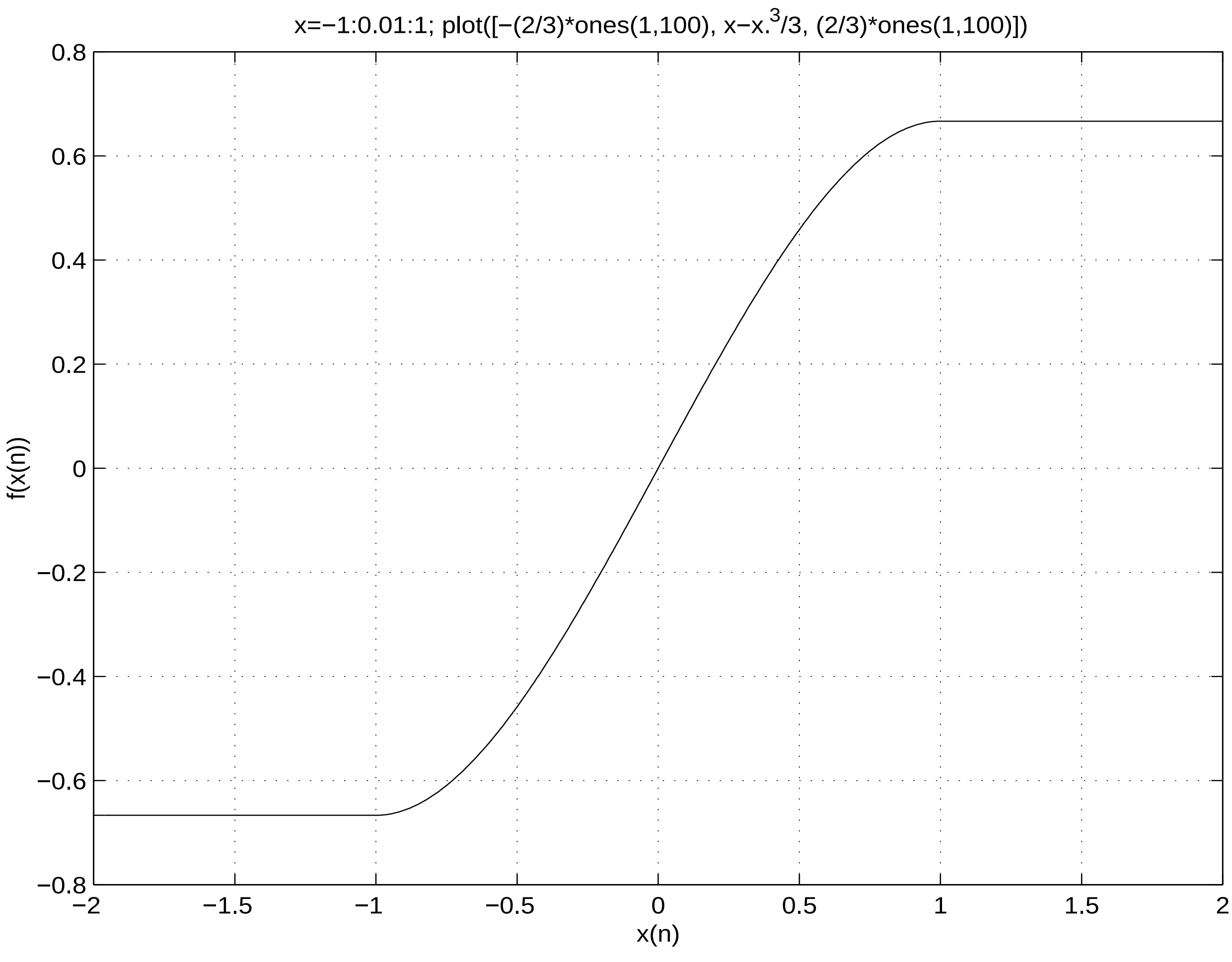
Without Dispersion

- Stacatto notes: (WAV) (MP3)
- Bach's First Suite for Unaccompanied Cello: (WAV) (MP3)

Nonlinear "Overdrive"

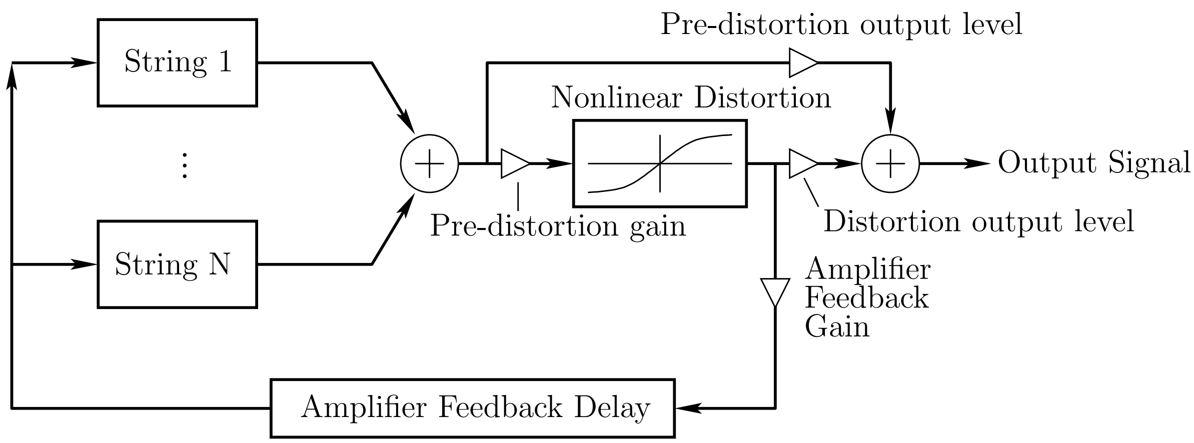
Soft Clipper

$$f(x) = \begin{cases} -\frac{2}{3}, & x \leq -1 \\ x - \frac{x^3}{3}, & -1 \leq x \leq 1 \\ \frac{2}{3}, & x \geq 1 \end{cases}$$



Amplifier Distortion + Amplifier Feedback

Sullivan 1990

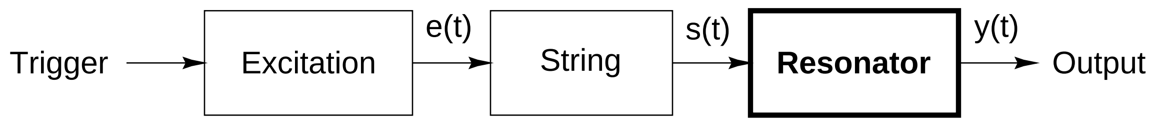


Distortion output signal often further filtered by an *amplifier cabinet filter*, representing speaker cabinet, driver responses, etc.

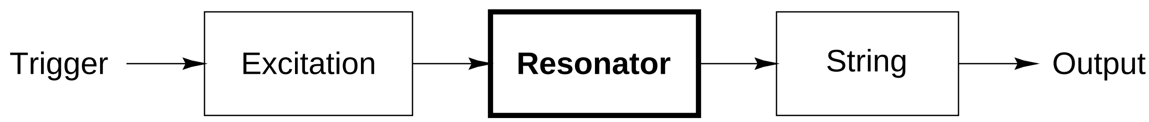
Sound Examples

- Distortion Guitar: (WAV) (MP3)
- Amplifier Feedback 1: (WAV) (MP3)
- Amplifier Feedback 2: (WAV) (MP3)

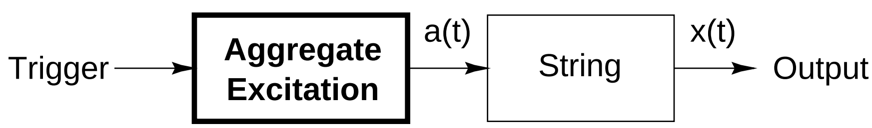
Commuted Synthesis of Acoustic Strings (1993)



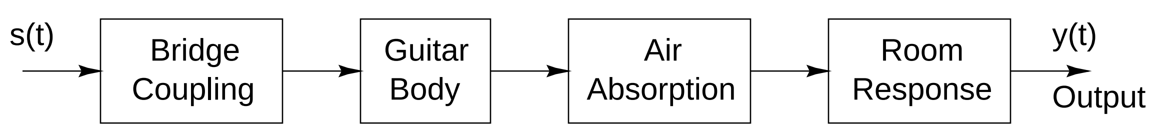
Schematic diagram of a stringed musical instrument.



Equivalent diagram in the linear, time-invariant case.



Use of an aggregate excitation given by the convolution of original excitation with the resonator impulse response.



Possible components of a guitar resonator.

Commuted Synthesis Sound Examples

Acoustic Guitar

- Bach Prelude in E Major: (AIFF) (MP3)
- Bach Loure in E Major: (AIFF) (MP3)

Virtual performance by Dr. Mikael Laurson[1], Sibelius Institute
Virtual guitar by Helsinki University of Technology, Acoustics Lab[2]

Electric Guitar (Pick-Ups and/or Body-Model Added)

- Example 1: (WAV) (MP3)
- Example 2: (WAV) (MP3)
- Example 3: (WAV) (MP3)
- Virtual "wah-wah pedal": (WAV) (MP3)

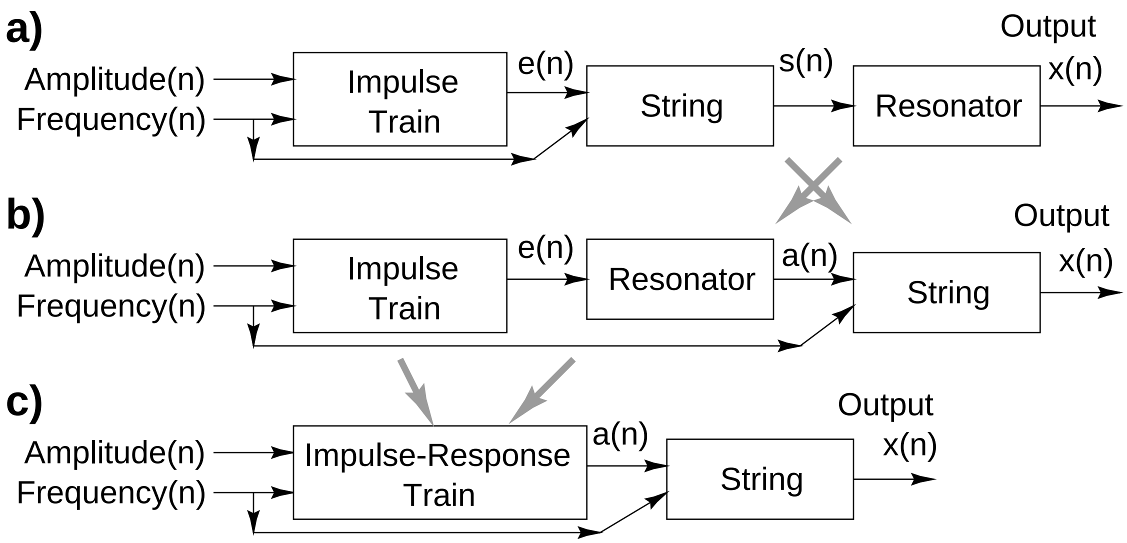
Stanford Sondius Project
Staccato Systems, Inc.

STK Mandolin

- STK Mandolin 1: (WAV) (MP3)
- STK Mandolin 2: (WAV) (MP3)

[1] http://www2.siba.fi/soundingscore/MikaelsHomePage/MikaelsHomepage.html
[2] http://www.acoustics.hut.fi/

Commuted Synthesis of Linearized Violin

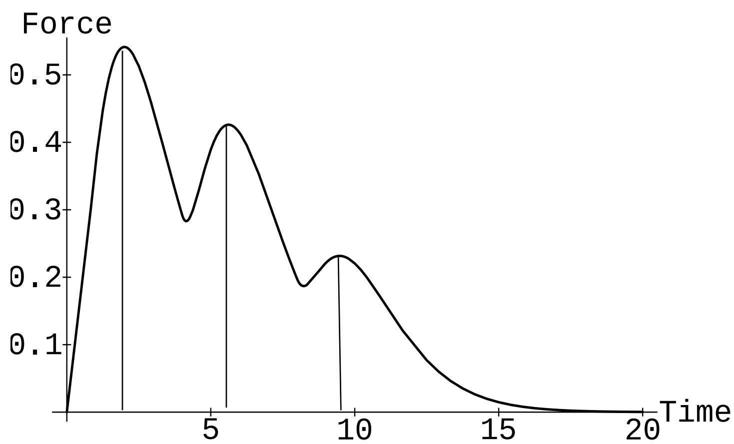


- Assumes *ideal Helmholtz motion* of string
- Sound Examples:
  - Double Bass: (WAV) (MP3)
  - Cello: (WAV) (MP3)
  - Viola 1: (WAV) (MP3)
  - Viola 2: (WAV) (MP3)
  - Violin 1: (WAV) (MP3)
  - Violin 2: (WAV) (MP3)
  - Ensemble: (WAV) (MP3)

Stanford Sondius Project
Staccato Systems, Inc.

Commuted Piano Synthesis (1995)

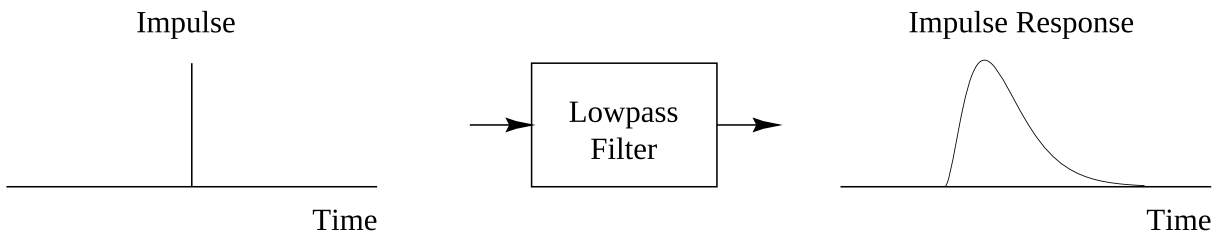
Hammer-string interaction pulses (force):



Vertical lines specify three *impulses* which will drive one to three *pulse-synthesis filters*

- Hammer = *mass* covered by *nonlinear spring* ("felt")
- String looks like a *resistor* upon initial impact
- Second and third pulses caused by *reflections* from agraffe (number depends on key number and hammer velocity)
- Pulses taller and thinner when hammer-velocity larger

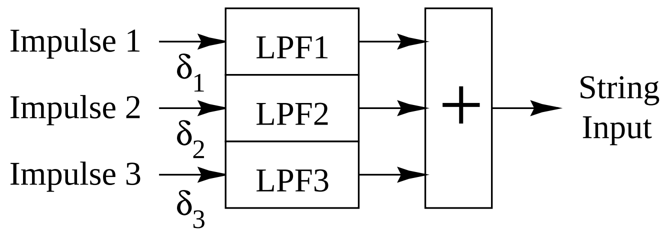
Synthesis of Hammer-String Interaction Pulse

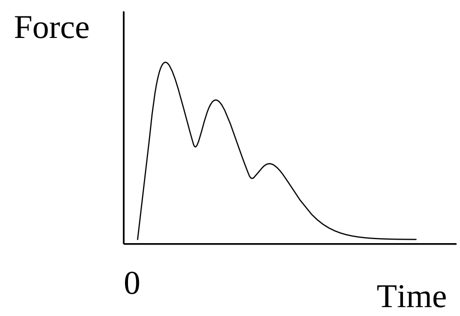


- Faster collisions correspond to *narrower* pulses (*nonlinear filter*)
- For a *given velocity*, filter is linear time-invariant
- Piano is "linearized" for each hammer velocity

Multiple Hammer-String Interaction Pulses

Superimpose several individual pulses:

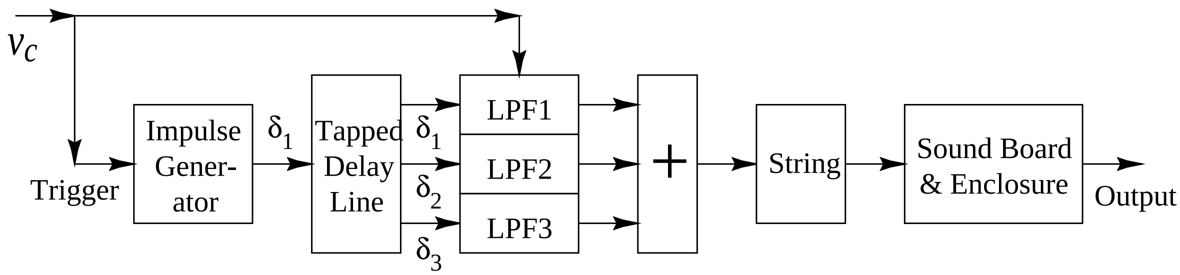




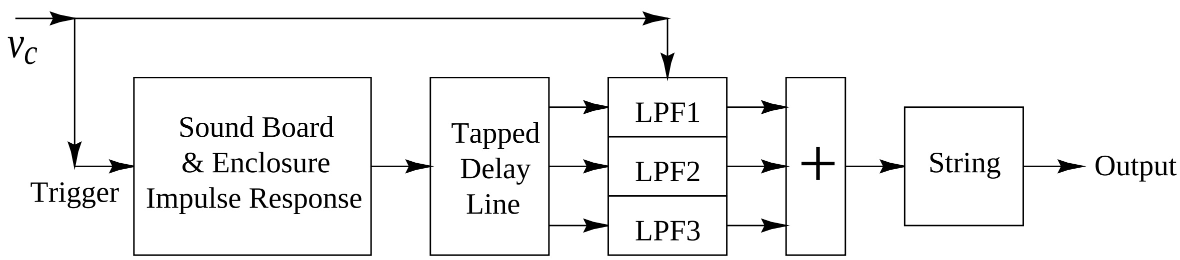
As impulse amplitude grows (faster hammer strike), output pulses become *taller and thinner*, showing less overlap.

Complete Piano Model

Natural Ordering:



Commuted Ordering:



- Soundboard and enclosure are *commuted*
- Only need a stored recording of their *impulse response*
- An enormous digital filter is otherwise required

Sound Examples

Piano and Harpsichord:

- Piano: (WAV) (MP3)
- Harpsichord 1: (WAV) (MP3)
- Harpsichord 2: (WAV) (MP3)

Stanford Sondius Project
Staccato Systems, Inc.

More Recent Harpsichord:

- Harpsichord Soundboard Hammer-Response: (WAV) (MP3)
- Musical Commuted Harpsichord Example: (WAV) (MP3)

Vesa Välimäki, Henri Penttinen, Jonte Knif, Mikael Laurson, and Cumhur Erkut
"Sound Synthesis of the Harpsichord Using a Computationally Efficient Physical Model", JASP-2004
`http://www.acoustics.hut.fi/publications/papers/jasp-harpsy/`
`Google` search: *Harpsichord Sound Synthesis*

Virtual Analog Synthesis

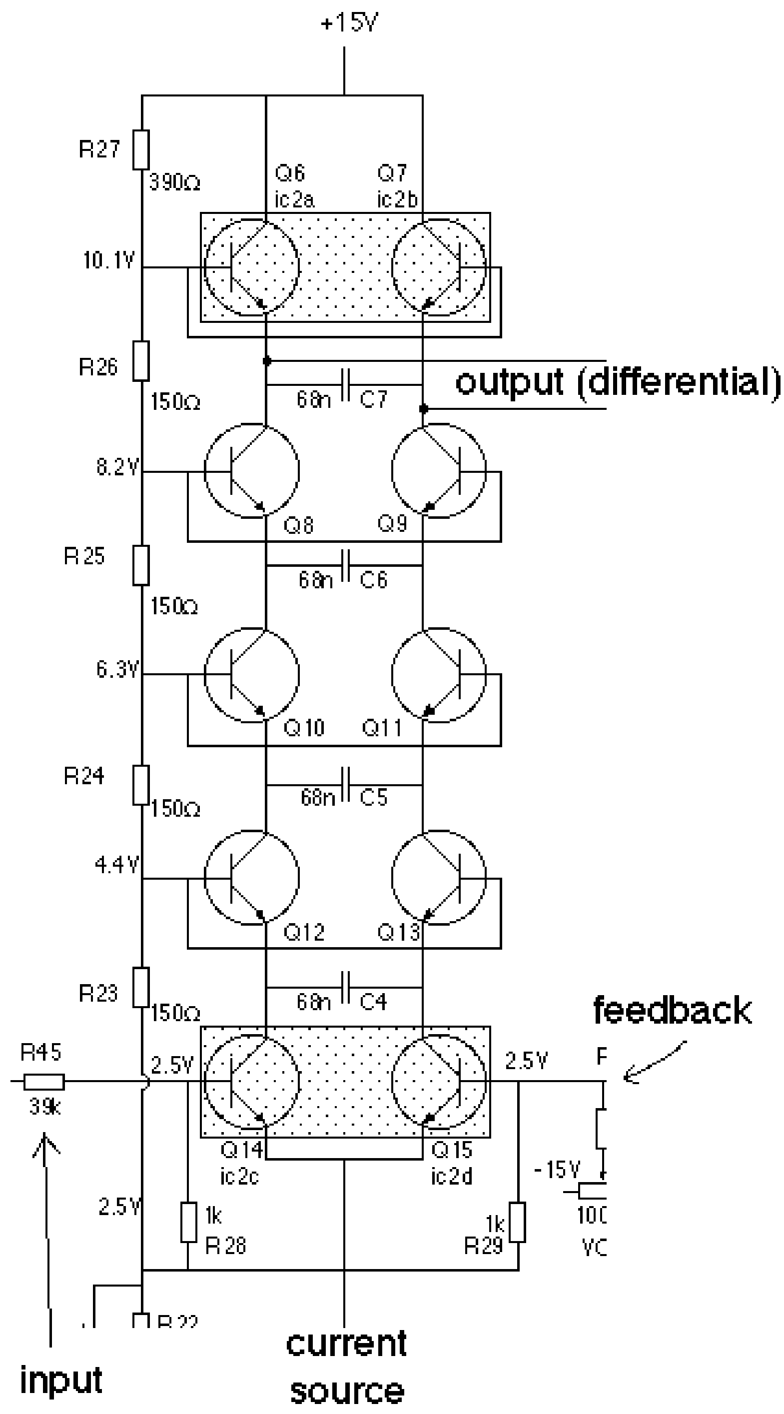
Most "Virtual Analog" synthesizers try to emulate some version of the MiniMoog or MemoryMoog synthesizers, because of their popularity. These classic synths were designed by the analog-synth pioneer Robert Moog.

Early Examples of Virtual Analog Synths:

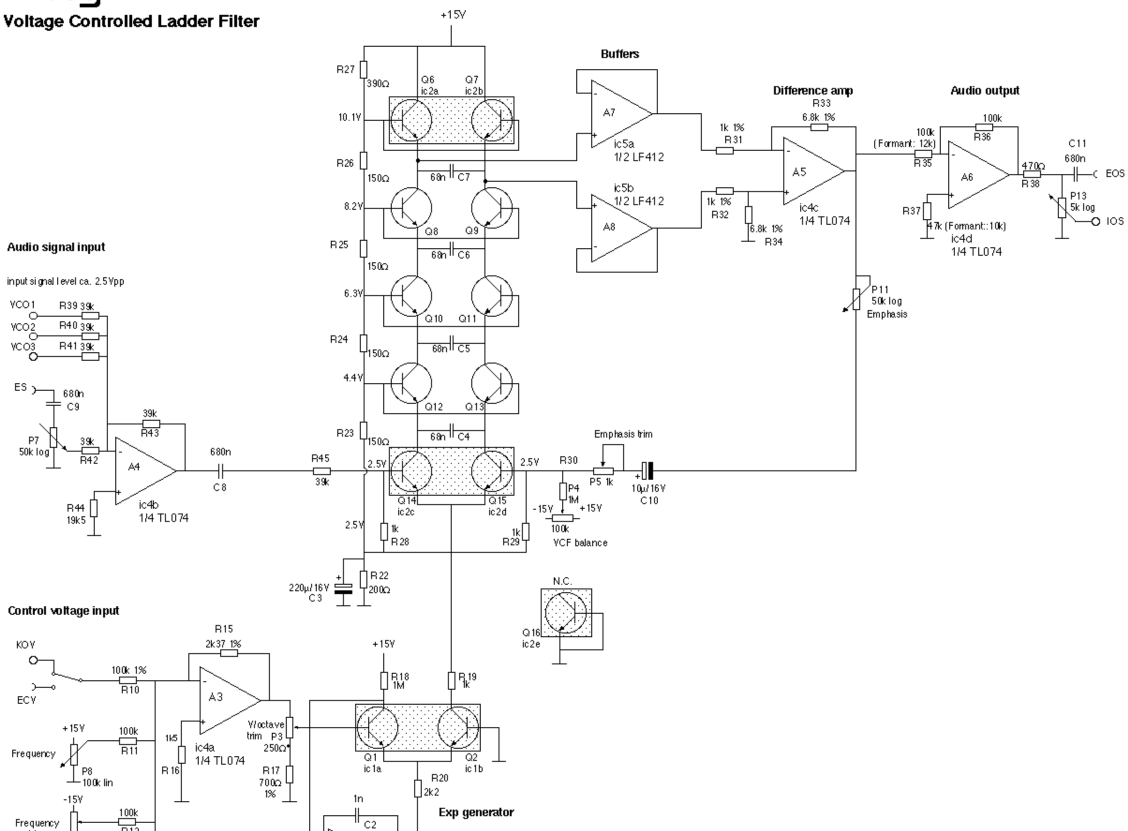
- Nordlead "Virtual Analog Synthesizer"
- Roland "Analog Modeling Synthesis"
- Yamaha "Analog Physical Modeling" (AN-1)

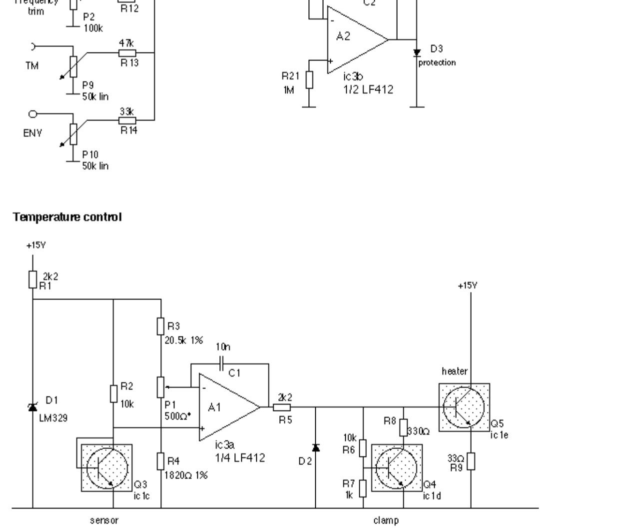
Design goal: Emulate the Moog Voltage Controlled Filter (VCF), due its popularity and excellent properties.

Moog VCF Ladder (1966)



Voltage Controlled Ladder Filter





R1,5,20 = 2k2
R2,6 = 10k
R3 = 20k5 1%
R4 = 1820Ω 1%
R7,19,28,29 = 1k
R8 = 330Ω
R9 = 33Ω
R10 = 100k 1%
R11,12,35,36 = 100k
R13,37 = 47k
R14 = 33k
R15 = 2k37 1%
R16 = 1k5
R17 = 700Ω 1%
R18,21,30 = 1M
R22 = 200Ω
R23,24,25,26 = 150Ω
R27 = 390Ω
R31,32 = 1k 1%
R33,34 = 6k8 1%
R38 = 470Ω
R39,40,41,42,43,45 = 39k
R44 = 19k5

Formant:
R35=12k
R37 = 10k

Q1-Q5 = IC1: LM3046 transistor array
Q6,Q7,Q14,Q15,Q16 = IC2: LM3046 transistor array
Q8,Q9,Q10,Q11,Q12,Q13 = BC109(a,b,c) low noise transistor
A1, A2 = IC3: LF412 (Low drift opamp)
A3,A4,A5,A6 = IC4: TL074
A7,A8 = IC5: LF412
D1 = IC6: LM329 (Precision low drift reference)
D2,D3 = 1N4148

C1 = 10n
C2 = 1n
C3 = 220μ/16V
C4,5,6,7 = 27n (prodigy) or 68n (mini)
C8,9,11 = 680n
C10 = 10μ/16V
C12,13,16,17,18,19 = 100n decoupling C
C14,15 = 470n..1μ decoupling C

P trim
P1 = 500Ω
P2,4 = 100k
P3 = 250Ω
P5 = 1k

Potentiometers
P7,11 = 50k log ES, Emphasis (Q)
P8 = 100k lin Frequency
P9,10 = 50k lin TM, ENV, VC Q
P13 = 5k log IOS (Internal Output Sigal)

2 x box10 header connector
DIN41617 31-pin connector

- All resistors recommended metal film
- Cermet trimmers recommended (especially P2 and P3)
- Capacitors MKM or MKF recommended

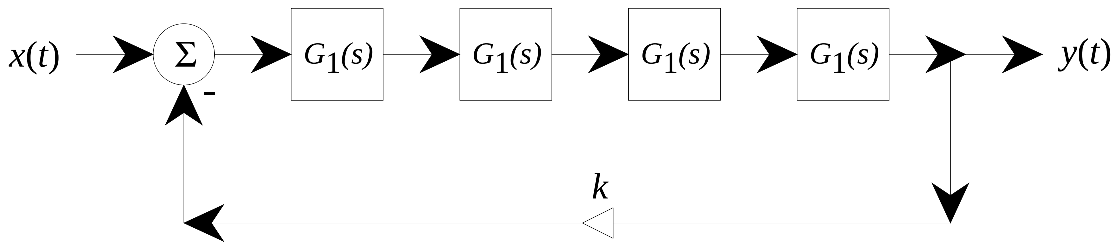
- The exp generator is inspired by the Elektor Formant VCO's exp generator
- Temperature control as in National Semiconductor Application Note AN-299 with added temperature adjustment.
- Pin-compatible with the Formant regular VCF

Revision 1.2
Last change: June 10th 1996

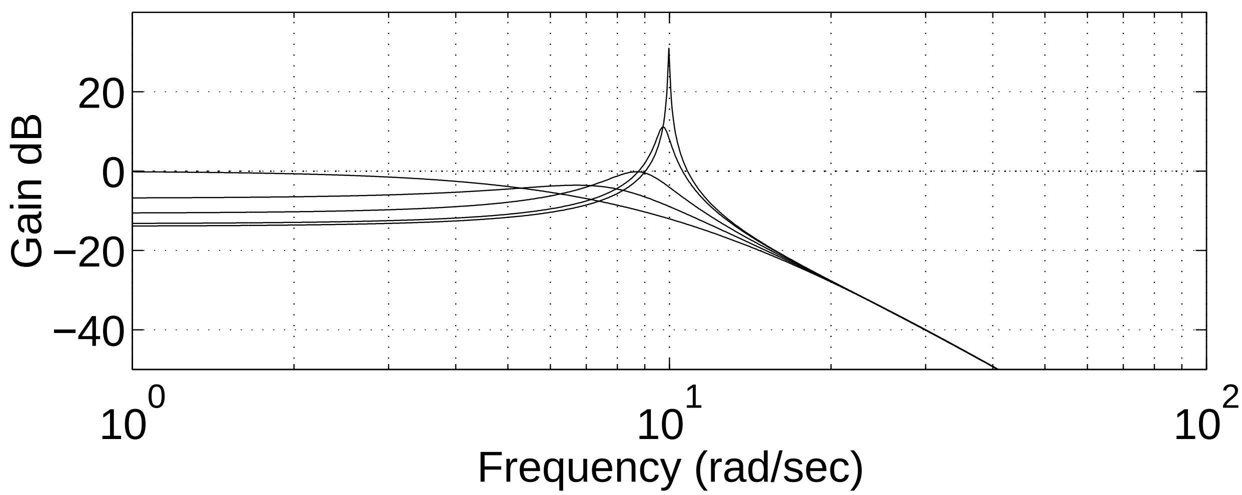
Many thanks to Don Tillman and Barry Klein for invaluable discussions.

The Moog VCF (1966)

Structure: four identical one-poles in series with a feedback loop:

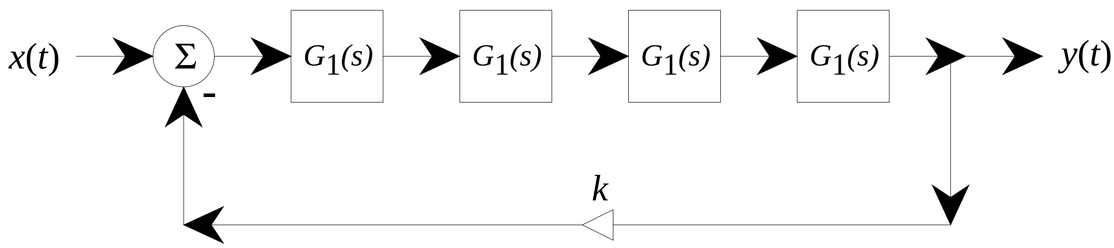


This implements a voltage-controllable four-pole filter:



In the Moog analog ladder (the "Moog 904A"), the one-pole filters consist of the capacitors and the AC resistance of the transistors, which is determined by the current source, which is varied to control tuning. See US Patent 3,475,623

Moog VCF Controls



Controls

- One-pole pole location: controls *cut-off frequency*
- Feedback gain: controls *resonance*

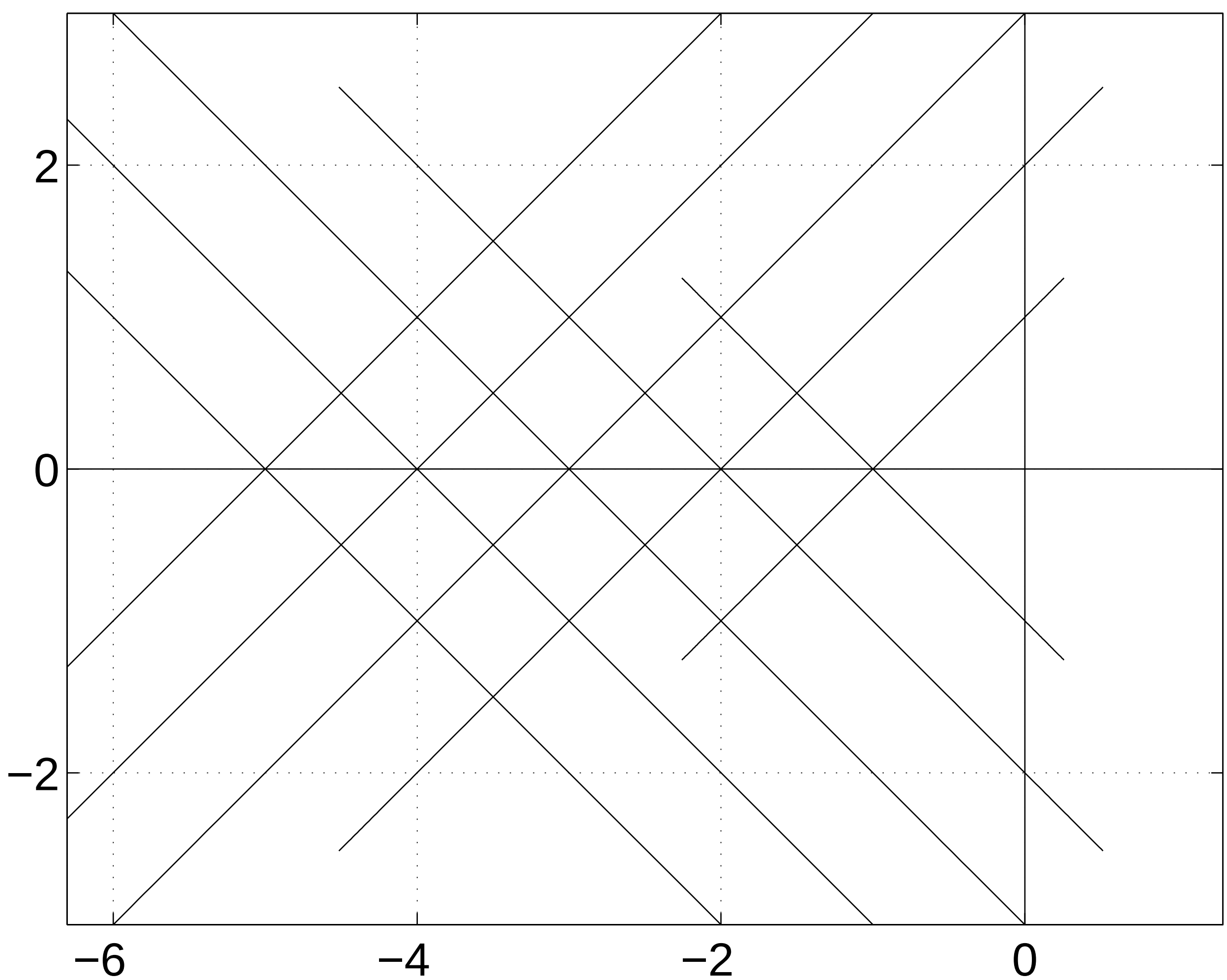
Resonance

- The phase of each pole at $s = -a$ is $45°$ when $s = ja$
- At this frequency ($\omega = a$), the phase through all four filters is $180°$
- The gain of each one-pole at $\omega = a$ is $1/\sqrt{2}$
  $\Rightarrow$ total gain is $1/4$
- Therefore, with a feedback gain of $k = 4$, the loop has *in-phase, positive feedback* at frequency $\omega = a$

Moog VCF Root Locus

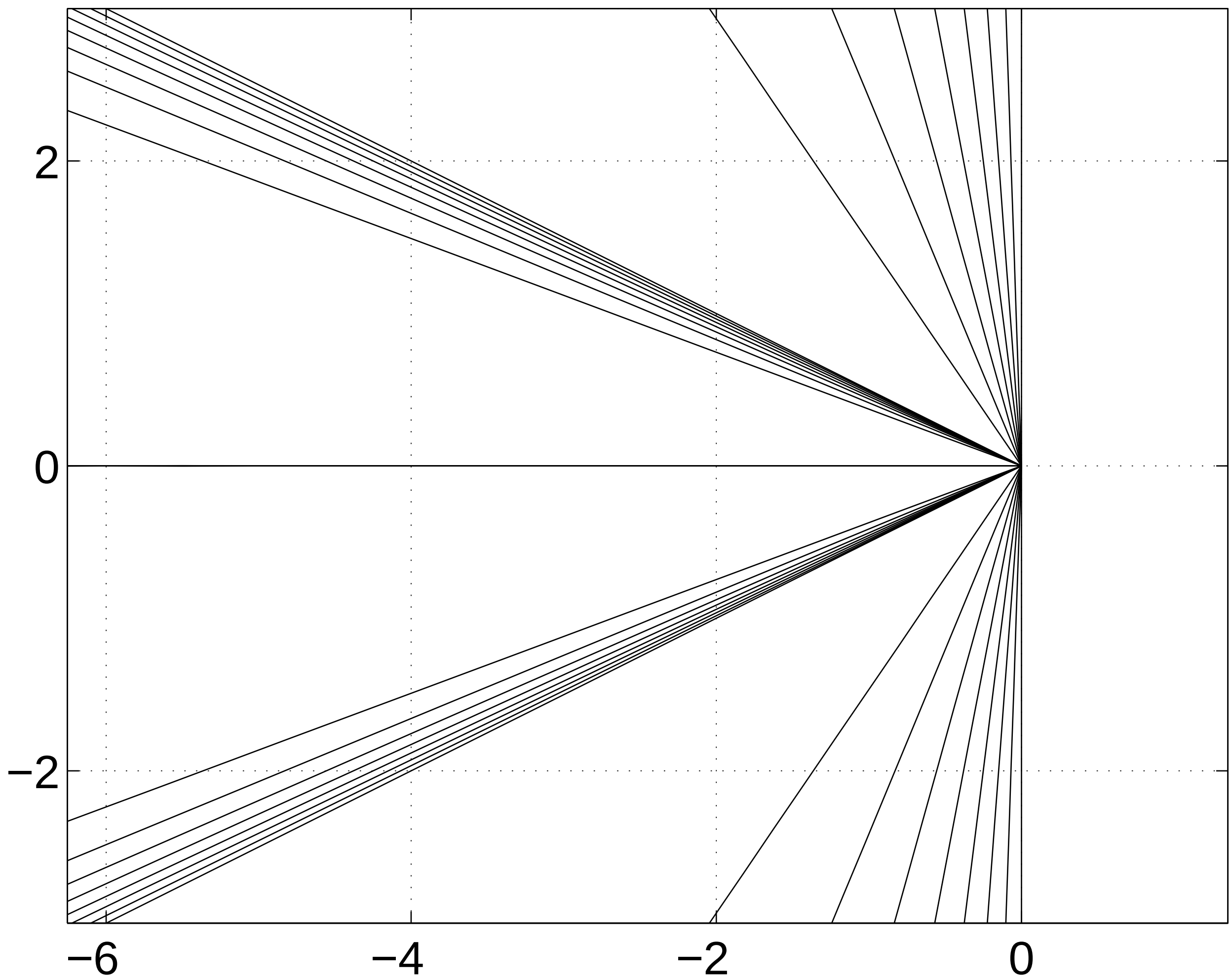
Some root loci of the continuous-time Moog VCF:

Root locus as k varies:



s plane

Root locus as one-pole pole location (p) varies:



s plane

Features of the Moog VCF

These loci show why the Moog VCF is such a good structure:

- Controls for cut-off and Q are completely orthogonal (constant-k contours are constant-Q contours)
- Controls are simply related to circuit parameters (resonance frequency $=$ open-loop poles)

Discrete-Time Moog VCF (1996)

Stilson and Smith, ICMC-96

Within the original structure (four one-poles in series with feedback around them), try various transforms from s to z:

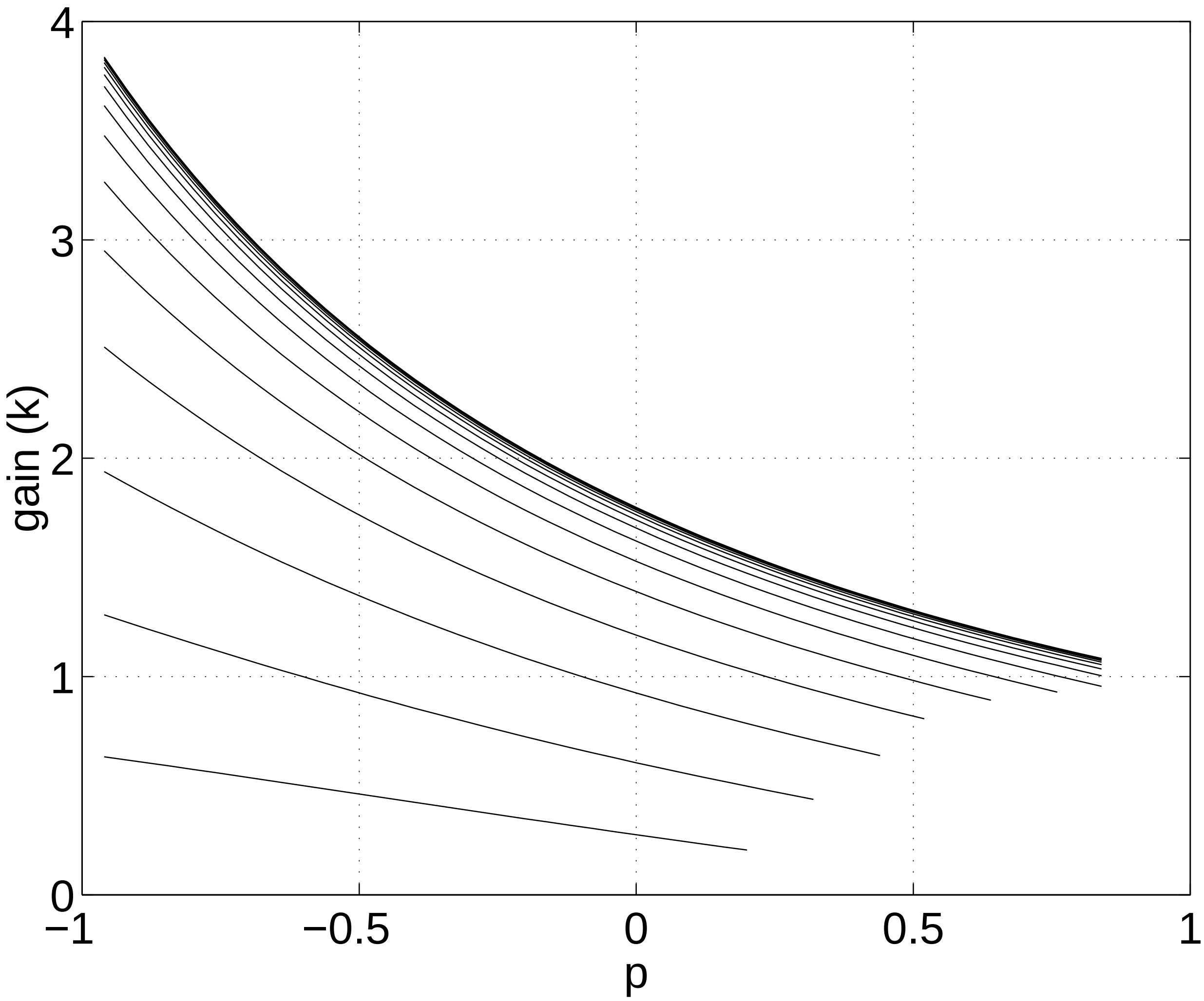
Backward difference: $G(z) = (p+1)z/(z+p)$

Bilinear: $G(z) = 0.5(p+1)(z+1)/(z+p)$

- **Problem:** Delay-free loops: These transforms cause the one-poles to have delay-free paths. Adding a delay to the loop changes the structure, so that complete orthogonality is no longer true (before the added delay, the bilinear case *was* orthogonal).
- *Separation tables* become necessary: The table we will use, in the bilinear-transform case, is a one-dimensional table that contains the feedback gain necessary to make the filter unstable, indexed by p. We scale k by this table ($k = \text{table}(p) k_{in}$).
- We would like to find a structure for which a separation table is unnecessary.

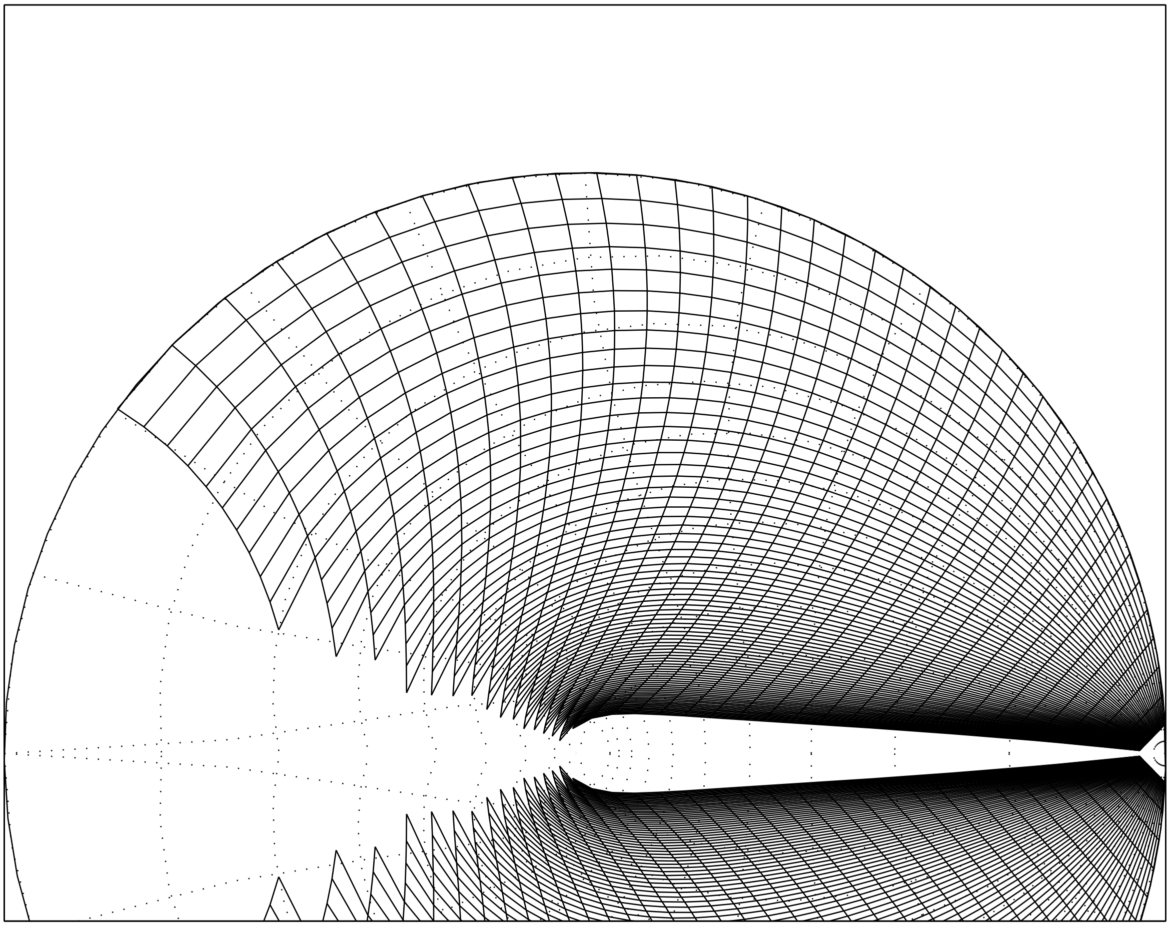
Separation Table

For the bilinear case, the separation table is the top curve:



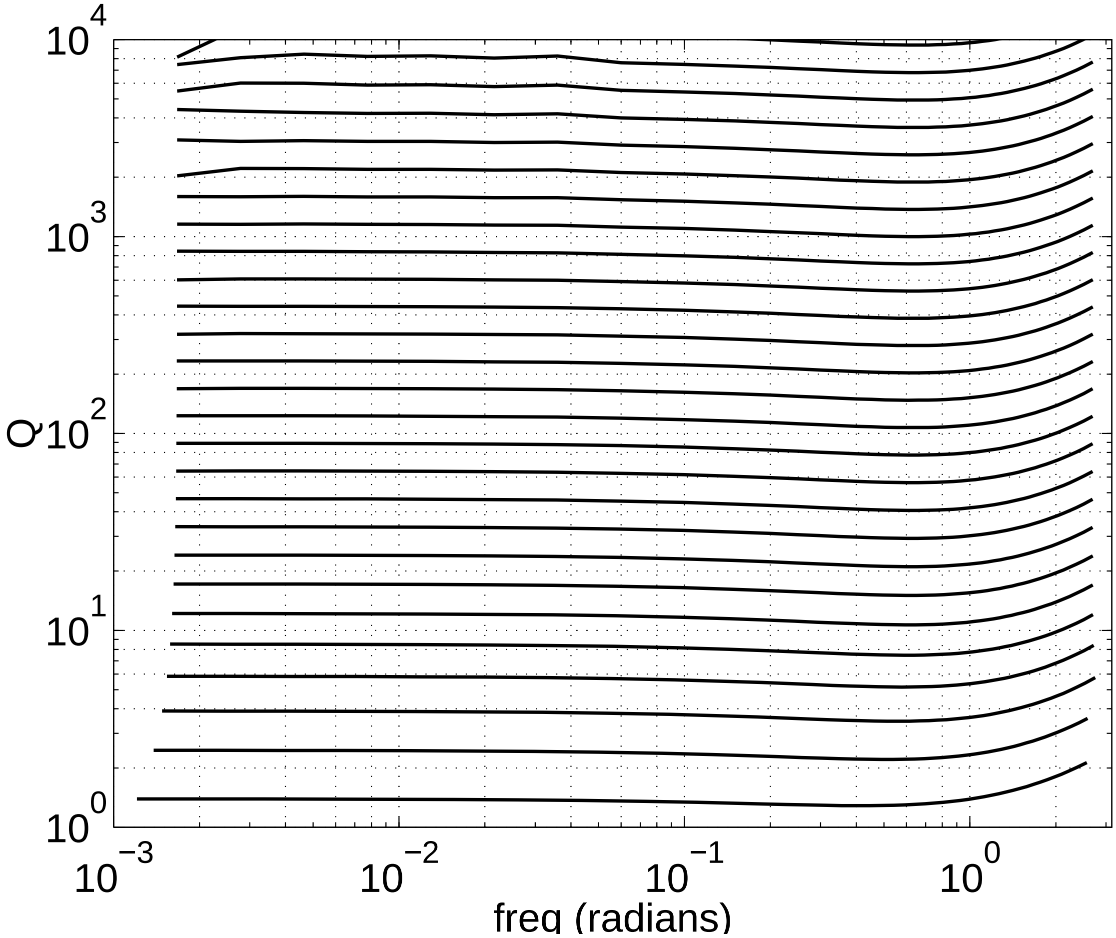
The other curves show gains for various values of Q (the top is infinite-Q).

Root Locus, Bilinear Transform Case (Using Separation Table)



Each curve generated by sweeping p with a fixed k

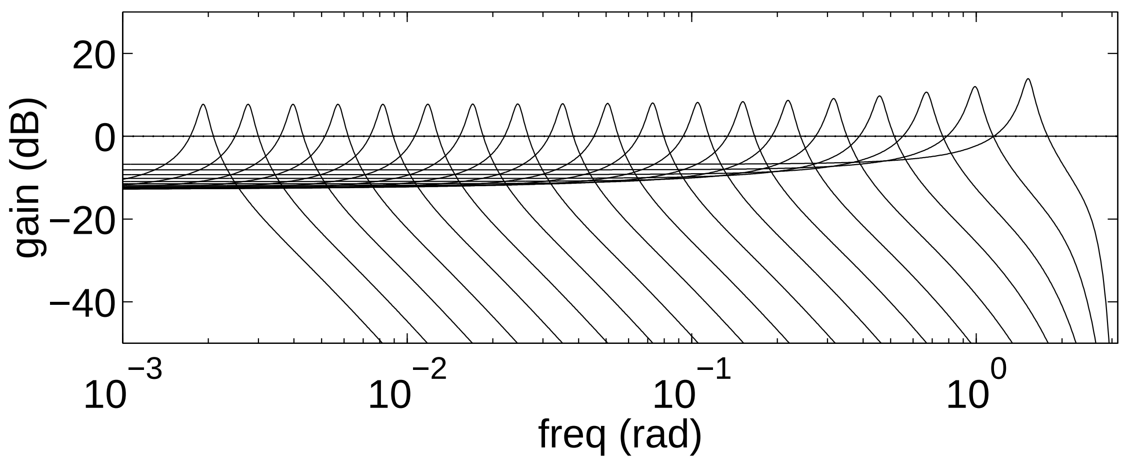
Q versus Cut-Off Frequency, Bilinear Transform, With Separation Table



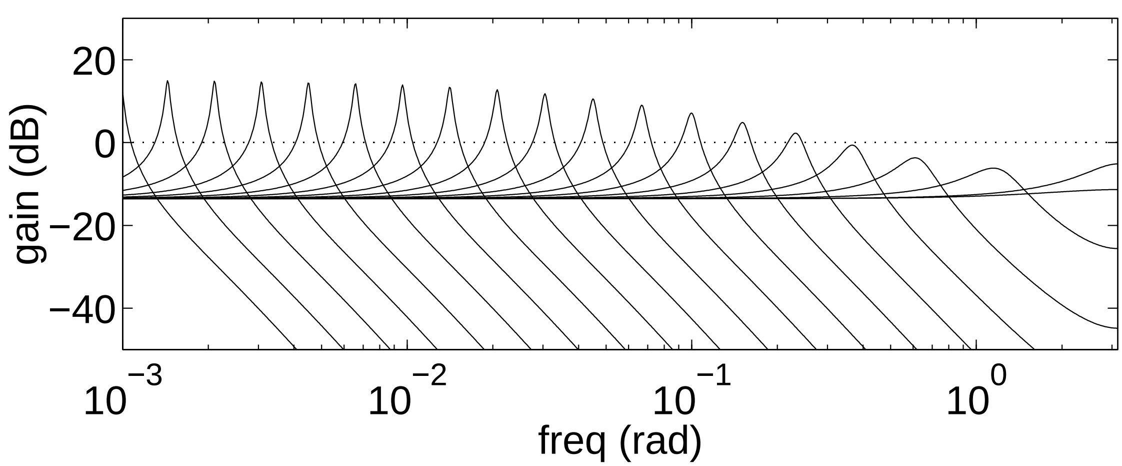
This case used the separation table for stability reasons.

Bode plots (medium Q)

Bilinear transform with separation table:



Backwards difference, no separation table:



Notes

- Constant-Q is hard for digital filters
  - Constant-Q tracks are logarithmic spirals
  - These aren't "root-locus primitives"
  - The separation table does a good job making constant-Q (didn't necessarily expect this)
- Look at zero locations between the two cases (remembering that we would like to get rid of the table). First, review the parameters we have to control:
  - easiest: the zero locations
  - also easy: the relative open-loop pole locations (so far, have only really looked at all equal)
  - more expensive: frequency-to-pole and Q-to-feedback-gain mapping functions (or more esoteric mappings)

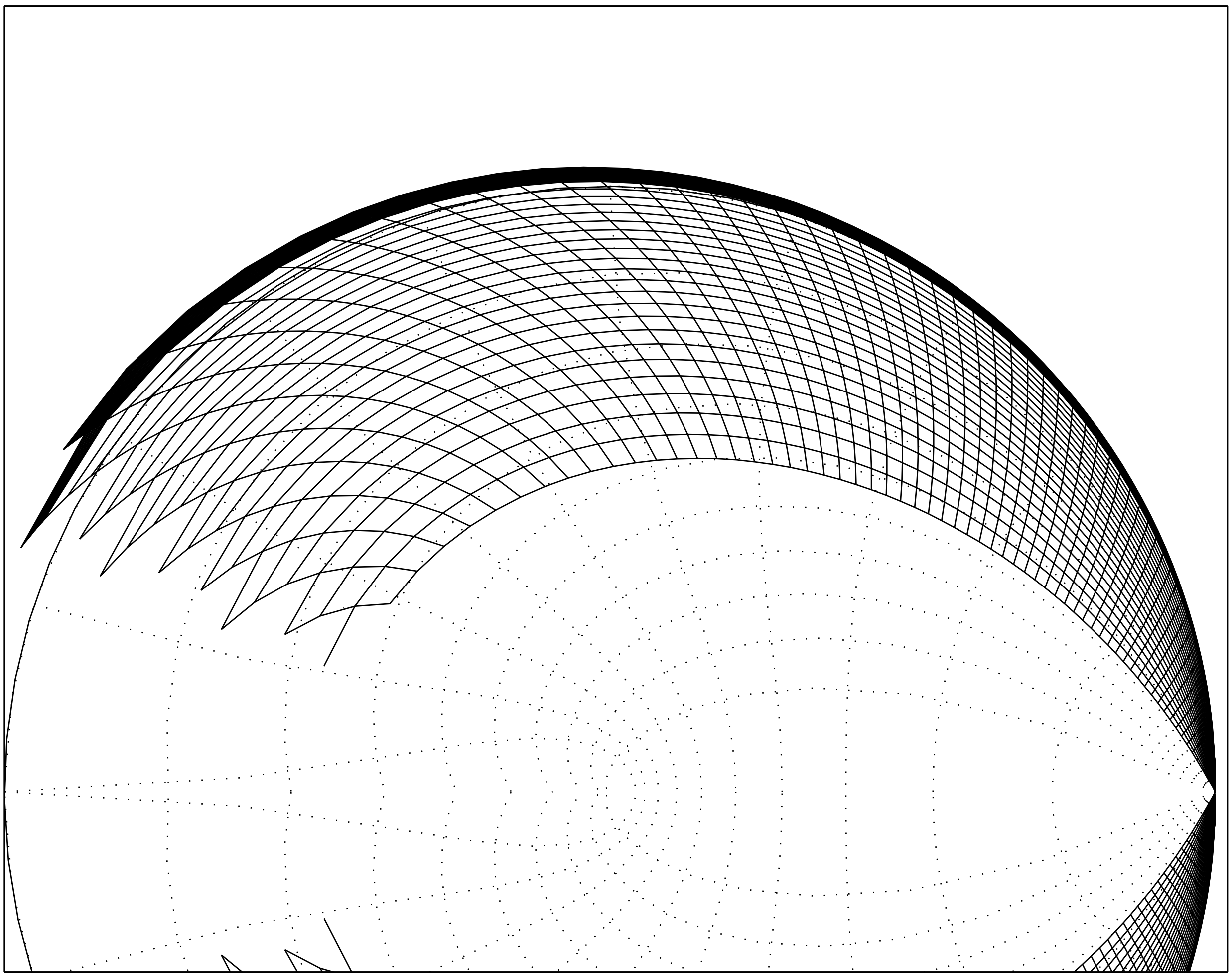
Note that

- backwards-difference yields zeros at $z = 0$
- bilinear transform yields zeros at $z = -1$

Try other locations...

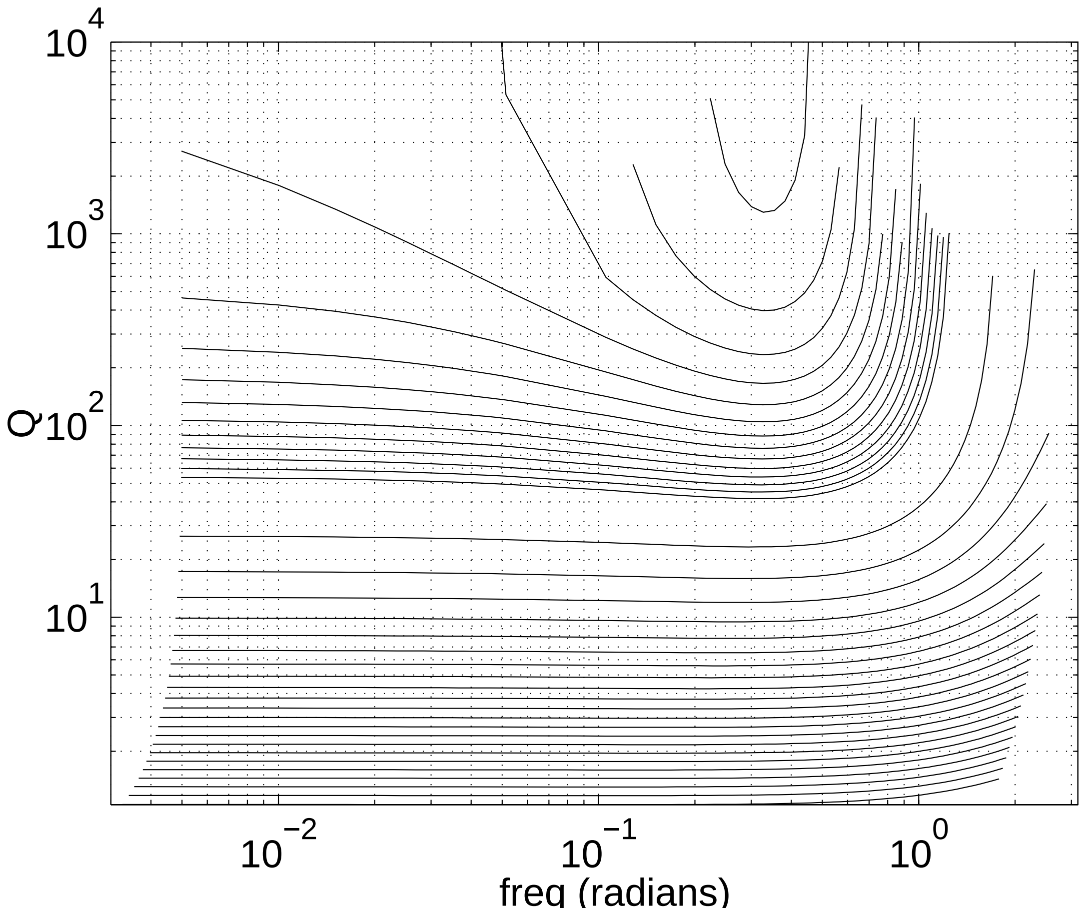
Discovery: There is a very good choice

Zeros at $z = 0.3$
(Tim Stilson 1996)



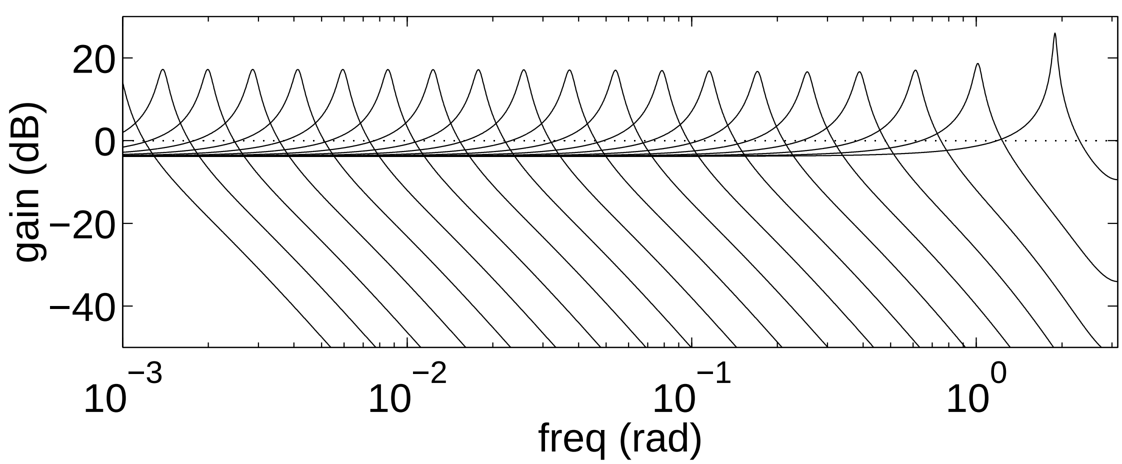
Root Locus vs. Tuning

Zeros at $z = 0.3$ — Q vs. Tuning



- Note that the Q-vs-frequency curves are pretty flat for $Q < 100$ for cut-off freq.s over most of the range
- This is quite good for not using a separation table

Zeros at $z = 0.3$ — Bode Plot



Farewell Bob Moog—-and Thank You!

Robert A. Moog (1934–2005)

Online Resources

- This presentation:
  http://ccrma.stanford.edu/~jos/Mohonk05/
- Book chapter from which the proceedings paper was condensed:
  http://ccrma.stanford.edu/~jos/pasp/-
  History_Enabling_Ideas.html