Physical Modelling of Musical Instruments Using Digital Waveguides:

History, Theory, Practice
Introduction

• Why Physical Modelling?
• History of Waveguide Physical Models
• Mathematics of Waveguide Physical Models, via Data Flow Diagrams
• Demonstration of Yamaha VL synthesizer
• Why Physical Modelling?
What Physical Modelling is NOT

- NOT looped playback of previously recorded sound (e.g. samplers)
What Physical Modelling is NOT

- NOT looped playback of previously recorded sound (e.g. samplers)
What Physical Modelling is NOT

- NOT synthesis by *ad hoc* similarity of sound (e.g. subtractive synthesis)
What Physical Modelling is
NOT

• NOT synthesis by *ad hoc* similarity of sound (e.g. subtractive synthesis)
What IS Physical Modelling?

- computer sound synthesis based on physical theory of the vibrating object - string, reed & air column, membrane

modes of vibration for a cylindrical air column

computer model of vibrating drum head
What IS Physical Modelling?

• analogous to computer graphics modelling of physical structures to simulate realistic dynamical behaviour

- animated dinosaur moves realistically by careful simulation of physical structure
- smoke synthesized with physical modelling (particle system)
What IS Physical Modelling?

- colliding galaxies synthesized by John Dubinski, U. of T. Dept. of Astronomy & Astrophysics
What IS Physical Modelling?

- simulates instrumental dynamics (i.e. behaviour)

  both desirable behaviour....
What IS Physical Modelling?

- simulates instrumental dynamics (i.e. behaviour)

both desirable behaviour....

Violin physical model, staccato
What IS Physical Modelling?

• simulates instrumental dynamics (i.e. behavior)

and undesirable behaviour....
What IS Physical Modelling?

- simulates instrumental dynamics (i.e. behaviour)

and undesirable behaviour....

Violin physical model, bowing overpressure
A More Complete Derivation of the String Wave Equation

Consider an elastic string under tension which is at rest along the $x$ dimension. Let $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ denote the unit vectors in the $x$, $y$, and $z$ directions, respectively. When a wave is present, a point $\mathbf{p} = (x, 0, 0)$ originally at $x$ along the string is displaced to some point $\mathbf{a} = \mathbf{p} + d\mathbf{p}$ specified by the displacement vector

$$d\mathbf{p} = i\zeta + j\eta + k\zeta'.$$

Note that typical derivations of the wave equation consider only the displacement $\eta$ in the $y$ direction. This more general treatment is adapted from [118].

The displacement of a neighboring point originally at $\mathbf{q} = (x + dx, 0, 0)$ along the string can be specified as

$$d\mathbf{q} = i(\zeta + d\zeta) + j(\eta + d\eta) + k(\zeta' + d\zeta').$$

Let $K$ denote string tension along $x$ when the string is at rest, and $\mathbf{K}$ denote the vector tension at the point $\mathbf{p}$ in the present displaced scenario under analysis. The net vector force acting on the infinitesimal string element between points $\mathbf{p}$ and $\mathbf{q}$ is given by the vector sum of the force $-\mathbf{K}$ at $\mathbf{p}$ and the force $K + (\partial K / \partial x)dx$ at $\mathbf{q}$, that is, $(\partial K / \partial x)dx$. If the string has stiffness, the two forces will in general not be tangent to the string at these points. The mass of the infinitesimal string element is $\epsilon dx$, where $\epsilon$ denotes the mass per unit length of the string at rest. Applying Newton’s second law gives

$$\frac{\partial \mathbf{K}}{\partial x} = \epsilon \frac{\partial^2 \mathbf{p}}{\partial x^2}$$

where $dx$ has been canceled on both sides of the equation. Note that no approximations have been made so far.

The next step is to express the force $\mathbf{K}$ in terms of the tension $K$, of the string at rest, the elastic constant of the string, and geometrical factors. The displaced string element $\mathbf{pq}$ is the vector

$$d\mathbf{s} = i(dx + d\zeta) + j\eta d\zeta + k\zeta' dx$$

having magnitude

$$d\mathbf{s} = \sqrt{\left(1 + \frac{\partial \zeta}{\partial x}\right)^2 + (\frac{\partial \eta}{\partial x})^2 + (\frac{\partial \zeta'}{\partial x})^2}.$$
Karplus-Strong Synthesis

- named after Stanford grad. students Kevin Karplus and Alex Strong

- efficient method for 8-bit microprocessor

- fill wavetable with random numbers

- average successive samples each time through the loop

  - averaging amounts to:
    - low-pass filtering (frequency domain description)
    - waveform smoothing (time domain description)
MidiForth
Karplus Workbench

Steady State Random Noise
Internal Low Pass Filter (LPF)
Varying Pitch
Karplus-Strong Synthesis


*HeatNoise* is a fantasy fantasy on the inter-relationship of signal and noise, meaning and error, chaos and order. Noise - taken broadly and metaphorically as the absence of meaning - and the emergence of meaning from noise is presented with sounds synthesized by means of the same fractal process used to generate the structure of the work; with the noisy sounds of speech, the sibilants, plosives and fricatives without which language would be unintelligible; with radio transmissions, including Neil Armstrong's famous non sequitur at the first moon landing; and with sounds, musical and otherwise, that employ noise in various ways to communicate a message.

Out of the opening chaos through the progressively greater disturbances of the underlying order, noise overwhelms meaning until we arrive at the place where the lost messages end - the radio transmissions that were never received, the cries for help that were never heard, the final gasps of those who died alone... Curiously enough, just as researchers in information theory found that indelicate four letter words were the first to emerge from the chaos of random letter orderings, so here we discover that the last sound to be heard as the chaos engulfs us is not profanity but... rock music.

*HeatNoise* is one of a series of algorithmic compositions applying principles of fractal geometry to music. The structural foundation for the work is an extended rhythmic figure generated by the fractal equation used to describe errors due to thermal noise encountered in data transmission.
**Karplus-Strong Synthesis**


*HeatNoise* is a fantasy on the inter-relationship of signal and noise, meaning and error, chaos and order. Noise - taken broadly and metaphorically as the absence of meaning - and the emergence of meaning from noise is presented with sounds synthesized by means of the same fractal process used to generate the structure of the work; with the noisy sounds of speech, the sibilants, plosives and fricatives without which language would be unintelligible; with radio transmissions, including Neil Armstrong's famous non sequitur at the first moon landing; and with sounds, musical and otherwise, that employ noise in various ways to communicate a message.

Out of the opening chaos through the progressively greater disturbances of the underlying order, noise overwhelms meaning until we arrive at the place where the lost messages end - the radio transmissions that were never received, the cries for help that were never heard, the final gasps of those who died alone... Curiously enough, just as researchers in information theory found that indelicate four letter words were the first to emerge from the chaos of random letter orderings, so here we discover that the last sound to be heard as the chaos engulfs us is not profanity but... rock music.

*HeatNoise* is one of a series of algorithmic compositions applying principles of fractal geometry to music. The structural foundation for the work is an extended rhythmic figure generated by the fractal equation used to describe errors due to thermal noise encountered in data transmission.
Stanford Prof. Julius O. Smith realized that this looped wavetable was equivalent to a digital representation of a vibrating string (or air column).

- developed the theoretical basis for what later became Waveguide Synthesis

- patented by Stanford in 1989 and licensed to Yamaha in 1994

- Yamaha’s previous licensing relationship with Stanford included FM Synthesis, which resulted in the best selling synthesizer of the period, (Yamaha DX7) and the 2nd most lucrative licensing agreement in Stanford’s history ($20,000,000)
MidiForth
Waveguide Workbench

Pluck Position
Bridge LPF
Varying Pickup Position
Varying Pitch
1.1. The Clarinet

Figure 1. Model of a single-reed, cylindrical-bore woodwind.

JOS Proposed Clarinet Model (1991)

Proposed Clarinet Implementation

\[
P_b^- = \rho \left( \frac{P^+_\Delta}{2} \right) \frac{P^+_\Delta}{2} + \frac{P_m}{2}
\]
JOS Proposed Clarinet Model (1991)

Clarinet Bore = Digital Waveguide (Bi-Directional Delay Line)

Right-Going Traveling Pressure Waves -->

Reed

Bell

<-- Left-Going Traveling Pressure Waves
Bell = Crossover Network

Crossover Wavelength = Bore Diameter

Bore

P⁺ → Highpass Filter → Bell Output

P⁻

Reflectance

Frequency

Transmittance

Freq. →
JOS Proposed Clarinet Model (1991)

Reed = Pressure-Controlled Valve

Bore sees a time-varying reflection-coefficient plus a time-varying mouth-pressure input

\[ P_b^- = \rho(P_\Delta) P_b^+ + \frac{1 - \rho(P_\Delta)}{2} P_m \]

\[ P_\Delta = P_b - P_m \]

\[ P_b = P_b^+ + P_b^- \]
1.2. The Bowed String

Figure 2. Basic model for a bowed string.

Yamaha VL1 Synthesizer

- Julius Smith & Stanford U. patented these techniques in late 1980s
- Yamaha licensed the patent in early ‘90s
- VL1 synthesizer was the first product of Yamaha’s license of Stanford’s waveguide technology
- 2 voices, 48 kHz, 16 bit
- Optimized for simulation of woodwind and brass instruments, esp. saxophones
## VL1 Architecture

<table>
<thead>
<tr>
<th>DRIVER</th>
<th>RESONATOR</th>
<th>modifiers</th>
</tr>
</thead>
<tbody>
<tr>
<td>WW</td>
<td>reed</td>
<td>Pipe</td>
</tr>
<tr>
<td>Brass</td>
<td>lips</td>
<td>Pipe</td>
</tr>
<tr>
<td>String</td>
<td>bow</td>
<td>String</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Driver</th>
<th>Resonator</th>
<th>Modifier</th>
</tr>
</thead>
<tbody>
<tr>
<td>WW</td>
<td>reed</td>
<td>Pipe</td>
</tr>
<tr>
<td>Brass</td>
<td>lips</td>
<td>Pipe</td>
</tr>
<tr>
<td>String</td>
<td>bow</td>
<td>String</td>
</tr>
</tbody>
</table>

- **WW**: reed, Pipe, Bell
- **Brass**: lips, Pipe, Bell
- **String**: bow, String, Body
### VL1 Pipe Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight Horn Insertion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Straight Horn1 Length</td>
<td>0</td>
<td>0.021ms</td>
</tr>
<tr>
<td>Straight 2/Conical Len</td>
<td>0</td>
<td>0.021ms</td>
</tr>
<tr>
<td>Delay Mode:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio 1 to 2</td>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>Ratio 2 to Conical</td>
<td>0</td>
<td>1.000</td>
</tr>
<tr>
<td>Short Length/Ratio</td>
<td>178</td>
<td></td>
</tr>
<tr>
<td>High Freq Abs. Mode</td>
<td>Both [-12 db/oct]</td>
<td></td>
</tr>
<tr>
<td>Damping/Decay</td>
<td>112</td>
<td>112</td>
</tr>
<tr>
<td>Register Key Open</td>
<td>74</td>
<td>D 5</td>
</tr>
<tr>
<td>Absorption</td>
<td>118</td>
<td>15000 h</td>
</tr>
<tr>
<td>1st Harmonic Dampening (Low/High Balance)</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>
VL1 - Tube Shape

Effect of Absorption on shape of Tube & Bell
Mozart – Clarinet Quintet

W. A. Mozart (1756-1791)
Imitative Synthesis 1 - Clarinet

Mozart – Clarinet Quintet

Allegro.

W. A. Mozart (1756-1791)
Imitative Synthesis 1 - Clarinet

Instrumental Behaviour:

- note transitions: legato vs tongued
- harmonic overblowing
Imitative Synthesis 2 - Oboe

- J.S. Bach BWV 1060 - Double Concerto

-all instruments are physically modelled except harpsichord
Imitative Synthesis 2 - Oboe

- J.S. Bach BWV 1060 - Double Concerto

-all instruments are physically modelled except harpsichord
Imitative Synthesis 2 - French Horn

Strauss – Till Eulenspiegel, op. 28
Instrumental Behavior:

The Harmonic Series
Imitative Synthesis 2 - Bassoon

Instrumental Behaviour: Staccato - note onsets and endings

In the Hall of the Mountain King
from 'Peer Gynt'
Edward Grieg (1843-1907)
Imitative Synthesis 2 - Saxophone

Instrumental Behaviour:
Acoustic Detail - breath noise
Imitative Synthesis 2 - Saxophone

Ornithology

By Charlie Parker and Benny Harris

prg 035 - Desmond
Emulative Synthesis 2 - Trombone

from Ravel - Bolero
Imitative Synthesis 1 - Woodwind Quintet

- Malcolm Arnold - Sea Shanty #1, arranged for “Virtual” Woodwind Quintet
Imitative Synthesis 1 - Woodwind Quintet

- Malcolm Arnold - Sea Shanty #1, arranged for “Virtual” Woodwind Quintet
  - Flute
  - Oboe
  - Clarinet
  - French Horn
  - Bassoon
Imitative Synthesis
Plucked Strings

Instrumental Behaviour - Spanish Guitar - pluck vs gliss.
Conclusion


- all instruments (except piano) are physically modelled
  - plucked - bass guitar, violin pizz
  - struck - hand drum
  - wind - soprano sax, scat “voice”

- musical structure is algorithmically generated by a synthetic process based on analytical theories of Heinrich Schenker
Conclusion


- all instruments (except piano) are physically modelled
  - plucked - bass guitar, violin pizz
  - struck - hand drum
  - wind - soprano sax, scat “voice”

- musical structure is algorithmically generated by a synthetic process based on analytical theories of Heinrich Schenker
Why Physical Modelling?

- **Acoustically Detailed**
  - breath noise
  - harmonic series
  - onset transients & note transitions

- **Synthesis Parameters have real-world counterparts**
  - can be extended beyond real-world limits to explore “impossible” instruments

- **Playability**
  - responsive
  - expressive
  - realistic
  - unpredictable
Why Physical Modelling?

- **Elegance** - sound generated from first principles rather than through *ad hoc* accumulation of imitative features
Why Physical Modelling?

- **Elegance** - sound generated from first principles rather than through *ad hoc* accumulation of imitative features

- invokes a mystery - “The Unreasonable Efficacy of Mathematics in Explaining the Physical World” (Eugene Wigner, 1960, quantum physicist, Nobel prize winner)

\[
ds = i(dx + d\xi) + j d\eta + k d\zeta \\
= \left[ i \left( 1 + \frac{\partial \zeta}{\partial x} \right) + j \frac{\partial \eta}{\partial x} + k \frac{\partial \zeta}{\partial x} \right] dx
\]